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MEASUREMENT UNCERTAINTY EVALUATION OF OBJECT COORDINATES IN PLANE BY THE GONIOMETRIC METHOD

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Abstract
The features of the uncertainty evaluation of measuring the coordinates of an object in plane by the goniometric method (theta position fixing) are discussed. Measurement model are presented that relate objects coordinates in the local rectangular coordinate system with the angles found using goniometers. The model includes corrections for determining the location of base stations, and correction associated with inaccuracies in the location of stations to the north. Uncertainty budgets for measurements of rectangular coordinates are given, which can be the basis for creating software for automating the calculation of measurement uncertainties. The estimates of the expanded uncertainties are found by the kurtosis method.
Keywords: coordinate metrology; goniometric method; measurement uncertainty, kurtosis method

1. Introduction

The problem of an object coordinates in plane determining is widely used in geodesy, radio navigation, radio-, optical and acoustic locations [1–4]. Depending on the number of base stations (BS) used in this case (radar, optoelectronic or acoustic) and their capabilities, this problem is solved by various methods.

The theta fixing (goniometric method) [5], considered in this article, is limited to the use of only one RS containing a rangefinder and a goniometer (direction finder). It belongs to the active methods of location, since it requires radiation from the BS to determine the distance to the object.

The goniometry method (theta location determination) considered in this article requires the presence of at least two BSs containing only goniometers (direction finders). It refers to passive location methods, since the BS radiation is not required to determine the distance to the object.

The report considers the features of measurement uncertainty evaluation the coordinates of an object in a plane using the goniometric method.

2. Justification of Measurement Models

Fig. 1 shows a diagram of the implementation of coordinates measuring of an object on a plane using the goniometric method.

The axis *OY* is directed to the north, the axis *OX* is drawn so as to form a right-hand rectangular coordinate system. The diagram shows basic stations O_1 and O_2, located at points with coordinates (x_1, y_1) and (x_2, y_2), respectively. The object *P* has the desired coordinates (x, y).

From the points of locations of the first and second stations O_1 and O_2, the direction to the target is established by direction finders, i.e. its azimuths α_1 and α_2, which are counted in the direction of movement of the clockwise from the direction to the north.

These parameters are used to determine the local Cartesian coordinates of the object (x, y) in accordance with the equations [6]:

$$x = x_1 + L_1 \cos\alpha_1 , \quad (1)$$

$$y = y_1 + L_1 \sin\alpha_1 , \quad (2)$$

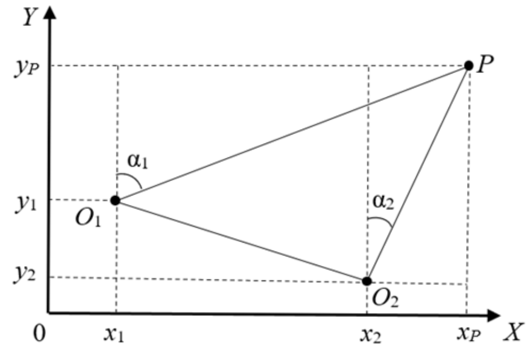

Fig. 1. Scheme of implementation of measurement of coordinates of an object on a plane by the goniometric method

where L_1 is the distance from the location of the first station point O_1 to point *P*, and:

$$L_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2} . \quad (3)$$

Similar expressions can be written for the second station:

$$x = x_2 + L_2 \cos\alpha_2 , \quad (4)$$

$$y = y_2 + L_2 \sin\alpha_2 \quad (5)$$

in which L_2 is the distance from the location of the second station point O_2 to the point *P*, and:

$$L_2 = \sqrt{(x - x_2)^2 + (y - y_2)^2} . \quad (6)$$

From Fig. 1 it can be seen that:

$$\left.\begin{aligned} x &= x_1 + (y - y_1)\text{ctg}\alpha_1 \\ x &= x_2 + (y - y_2)\text{ctg}\alpha_2 \end{aligned}\right\}. \quad (7)$$

Solving system (7) for y, we have:

$$y = \frac{(x_2 - x_1) + (y_1\text{ctg}\alpha_1 - y_2\text{ctg}\alpha_2)}{\text{ctg}\alpha_1 - \text{ctg}\alpha_2}. \quad (8)$$

Substituting this solution into the first equation of system (7), we obtain an expression for the x coordinate of the object:

$$x = x_1 + \text{ctg}\alpha_1 \cdot \left[\frac{(x_2 - x_1) + (y_1 - y_2)\text{ctg}\alpha_2}{\text{ctg}\alpha_1 - \text{ctg}\alpha_2}\right]. \quad (9)$$

Thus, the obtained equations (8) and (9) allow us to obtain the coordinates of the object (x, y) through the coordinates of the locations of the station points (x_1 , y_1) and (x_2 , y_2), as well as the target azimuths α_1 and α_2, measured by both stations.

3. Evaluation of Numerical Values and Measurement Uncertainties of Object

The numerical values of the measurands $\hat{x}, \hat{y}$ can be determined from expressions (8)-(9) by substituting the values of the input quantities $\hat{x}_1$, $\hat{y}_1$ $\hat{x}_2$, $\hat{y}_2$, $\hat{\alpha}_1$, $\hat{\alpha}_2$ into them:

$$\hat{x} = \hat{x}_1 + \text{ctg}\hat{\alpha}_1\left[\frac{(\hat{x}_2 - \hat{x}_1) + (\hat{y}_1 - \hat{y}_2)\text{ctg}\hat{\alpha}_2}{\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2}\right]; \quad (10)$$

$$\hat{y} = \frac{(\hat{x}_2 - \hat{x}_1) + (\hat{y}_1\text{ctg}\hat{\alpha}_1 - \hat{y}_2\text{ctg}\hat{\alpha}_2)}{\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2}. \quad (11)$$

According to the obtained measurement models (10), (11) of the object coordinates, in accordance with the rule of summation of variances [7], it is possible to write expressions for the standard uncertainties of their measurement.

For coordinate x:

$$u^2(\hat{x}) = \{[c_{x_1}(\hat{x})u(\hat{x}_1)]^2 + [c_{x_2}(\hat{x})u(\hat{x}_2)]^2 + [c_{y_1}(\hat{x})u(\hat{y}_1)]^2 + \\ + [c_{y_2}(\hat{x})u(\hat{y}_2)]^2 + [c_{\alpha_1}(\hat{x})u(\hat{\alpha}_1)]^2 + [c_{\alpha_2}(\hat{x})u(\hat{\alpha}_2)]^2\}^{0,5}, \quad (12)$$

where the corresponding sensitivity coefficients $c_{x_1}(\hat{x})$, $c_{x_2}(\hat{x})$, $c_{y_1}(\hat{x})$, $c_{y_2}(\hat{x})$, $c_{\alpha_1}(\hat{x})$ и $c_{\alpha_2}(\hat{x})$ are determined by the expressions:

$$c_{x_1}(\hat{x}) = \frac{\partial\hat{x}}{\partial\hat{x}_1} = \frac{-\text{ctg}\hat{\alpha}_1}{\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2}; \quad (13)$$

$$c_{x_2}(\hat{x}) = \frac{\partial\hat{x}}{\partial\hat{x}_2} = \frac{\text{ctg}\hat{\alpha}_1}{\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2}; \quad (14)$$

$$c_{y_1}(\hat{x}) = \frac{\partial\hat{x}}{\partial\hat{y}_1} = \frac{\text{ctg}\hat{\alpha}_1\text{ctg}\hat{\alpha}_2}{\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2}; \quad (15)$$

$$c_{y_2}(\hat{x}) = \frac{\partial\hat{x}}{\partial\hat{y}_2} = \frac{-\text{ctg}\hat{\alpha}_1\text{ctg}\hat{\alpha}_2}{\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2}; \quad (16)$$

$$c_{\alpha_1}(\hat{x}) = \frac{\partial\hat{x}}{\partial\hat{\alpha}_1} = \frac{\text{ctg}\hat{\alpha}_2\left[(\hat{x}_2 - \hat{x}_1) + \text{ctg}\hat{\alpha}_2(\hat{y}_1 - \hat{y}_2)\right]}{\left[(\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2)\sin\hat{\alpha}_1\right]^2}; \quad (17)$$

$$c_{\alpha 2}(\hat{x}) = \frac{\partial\hat{x}}{\partial\hat{\alpha}_2} = \frac{(\hat{y}_2 - \hat{y}_1)\text{ctg}\hat{\alpha}_1 + (\hat{x}_2 - \hat{x}_1)}{[(\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2)\sin\hat{\alpha}_2]^2}. \quad (18)$$

Similarly, we can write an expression for the standard measurement uncertainty of the coordinate y:

$$u(\hat{y}) = \{[c_{x_1}(\hat{y})u(\hat{x}_1)]^2 + [c_{x_2}(\hat{y})u(\hat{x}_2)]^2 + [c_{y_1}(\hat{y})u(\hat{y}_1)]^2 + \\ + [c_{y_2}(\hat{y})u(\hat{y}_2)]^2 + [c_{\alpha_1}(\hat{y})u(\hat{\alpha}_1)]^2 + [c_{\alpha_2}(\hat{y})u(\hat{\alpha}_2)]^2\}^{0,5}, \quad (19)$$

in which the corresponding sensitivity coefficients $c_{x_1}(\hat{y})$, $c_{x_2}(\hat{y})$, $c_{y_1}(\hat{y})$, $c_{y_2}(\hat{y})$, $c_{\alpha_1}(\hat{y})$ and $c_{\alpha_2}(\hat{y})$ are found as:

$$c_{x_1}(\hat{y}) = \frac{\partial\hat{y}}{\partial\hat{x}_1} = \frac{-1}{\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2}; \quad (20)$$

$$c_{x_2}(\hat{y}) = \frac{\partial\hat{y}}{\partial\hat{x}_2} = \frac{1}{\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2}; \quad (21)$$

$$c_{y_1}(\hat{y}) = \frac{\partial\hat{y}}{\partial\hat{y}_1} = \frac{\text{ctg}\hat{\alpha}_1}{\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2}; \quad (22)$$

$$c_{y_2}(\hat{y}) = \frac{\partial\hat{y}}{\partial\hat{y}_2} = \frac{-\text{ctg}\hat{\alpha}_2}{\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2}; \quad (23)$$

$$c_{\alpha_1}(\hat{y}) = \frac{\partial\hat{y}}{\partial\hat{\alpha}_1} = \frac{(\hat{x}_2 - \hat{x}_1) + (\hat{y}_1 - \hat{y}_2)\text{ctg}\hat{\alpha}_2}{\left[(\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2)\sin\hat{\alpha}_1\right]^2}, \quad (24)$$

$$c_{\alpha_2}(\hat{y}) = \frac{\partial\hat{y}}{\partial\hat{\alpha}_2} = \frac{(\hat{x}_1 - \hat{x}_2) + (\hat{y}_2 - \hat{y}_1)\text{ctg}\hat{\alpha}_1}{\left[(\text{ctg}\hat{\alpha}_1 - \text{ctg}\hat{\alpha}_2)\sin\hat{\alpha}_2\right]^2}. \quad (25)$$

In expressions (12) and (19) $u(\hat{\alpha}_1)$, $u(\hat{\alpha}_2)$ are the standard measurement uncertainties $\hat{\alpha}_1$, $\hat{\alpha}_2$, defined as:

$$u(\hat{\alpha}_1) = \sqrt{u_I^2(\hat{\alpha}_1) + u_N^2(\hat{\alpha}_1)}; \quad (26)$$

$$u(\hat{\alpha}_2) = \sqrt{u_I^2(\hat{\alpha}_2) + u_N^2(\hat{\alpha}_2)}, \quad (27)$$

where $u_I(\hat{\alpha}_1)$, $u_I(\alpha_2)$ are the standards instrumental measurement uncertainties of the angular coordinates $\hat{\alpha}_1$ and $\hat{\alpha}_1$ with the help of a goniometer; $u_N(\hat{\alpha}_1)$, $u_N(\hat{\alpha}_2)$ are the component of uncertainty associated with the inaccuracy of binding the BS to the direction to the north.

Standard uncertainties $u(\hat{x}_1)$, $u(\hat{y}_1)$ and $u(\hat{x}_2)$, $u(\hat{y}_2)$ determinations of the locations standing points O_1 and O_2, respectively, are found from the corresponding boundaries of the maximum permissible error $\pm\theta(\hat{x})$ $\pm\theta(\hat{y})$ under the assumption of an uniform distributions of the errors in the estimates $\hat{x}_1$, $\hat{y}_1$, $\hat{x}_2$, $\hat{y}_2$ within these boundaries:

$$u(\hat{x}_1) = u(\hat{x}_2) = \theta(\hat{x})\big/\sqrt{3}; \quad (28)$$

$$u(\hat{y}_1) = u(\hat{y}_2) = \theta(\hat{y})\big/\sqrt{3}. \quad (29)$$

Instrumental uncertainties in measuring angular coordinates α_1 and α_2 using goniometers $u_I(\hat{\alpha}_1)$, $u_I(\hat{\alpha}_2)$, which can be found through the boundaries of the maximum permissible instrumental error in measuring azimuth $\pm\hat{\theta}_\alpha$, assuming an uniform distribution of instrumental errors within these boundaries as:

$$u_I(\hat{\alpha}_1) = u_I(\hat{\alpha}_2) = \hat{\theta}_\alpha \big/ \sqrt{3}\,; \qquad (30)$$

If the limits of the maximum permissible error of reference to the direction to the north are taken to be equal to $\pm\hat{\theta}_N$, then, assuming an uniform distribution of the reference error within these limits, we can write:

$$u_N(\hat{\alpha}_1) = u_N(\hat{\alpha}_2) = \hat{\theta}_N \big/ \sqrt{3}\,. \qquad (31)$$

Since the standard uncertainties of all input quantities were determined according to type B and were assigned an uniform distribution, the expanded uncertainties of coordinate measurement (x, y) are best found using the kurtosis method [8]:

$$U(x) = k(\eta_x)u(x)\,; \qquad (32)$$

$$U(y) = k(\eta_y)u(y)\,, \qquad (33)$$

where the coverage factors for the confidence level of 0.95 and 0,9545 are found using the formula:

$$\left.\begin{aligned} k_{0,95} &= 0{,}1085\eta^3 + 0{,}1\eta + 1{,}96 \\ k_{0,9545} &= 0{,}12\eta^3 + 0{,}1\eta + 2{,}0 \end{aligned}\right\}, \qquad (34)$$

and the kurtosis of the distribution for (x, y) will be equal to:

$$\eta(x) = \frac{-1{,}2}{u^4(\hat{x})}\{[c_{x_1}(\hat{x})u(\hat{x}_1)]^4 + [c_{x_2}(\hat{x})u(\hat{x}_2)]^4 + [c_{y_1}(\hat{x})u(\hat{y}_1)]^4 + [c_{y_2}(\hat{x})u(\hat{y}_2)]^4 + [c_{\alpha_1}(\hat{x})u_I(\hat{\alpha}_1)]^4 + [c_{\alpha_1}(\hat{x})u_N(\hat{\alpha}_1)]^4 + [c_{\alpha_2}(\hat{x})u_I(\hat{\alpha}_2)]^4 + [c_{\alpha_2}(\hat{x})u_N(\hat{\alpha}_2)]^4\,; \qquad (35);$$

$$\eta(y) = \frac{-1{,}2}{u^4(\hat{y})}\{[c_{x_1}(\hat{y})u(\hat{x}_1)]^4 + [c_{x_2}(\hat{y})u(\hat{x}_2)]^4 + [c_{y_1}(\hat{y})u(\hat{y}_1)]^4 + [c_{y_2}(\hat{y})u(\hat{y}_2)]^4 + [c_{\alpha_1}(\hat{y})u_I(\hat{\alpha}_1)]^4 + [c_{\alpha_1}(\hat{y})u_N(\hat{\alpha}_1)]^4 + [c_{\alpha_2}(\hat{y})u_I(\hat{\alpha}_2)]^4 + [c_{\alpha_2}(\hat{y})u_N(\hat{\alpha}_2)]^4\,. \qquad (36)$$

Formulas (35), (36) take into account that the kurtosis of all input values that have an equally probable distribution are equal to (-1.2).

The uncertainty budgets of the object coordinate (x, y) measurements will have the form given in Tables 1 and 2.

Table 1 – Uncertainty budget for the measurement of the x-coordinate

Input quantities	**Values of input quantities**	**Standard uncertainties of input quantities**	**Kurtosis of input quantities**	**Sensitivity coefficients**	**Uncertainty contributions**
x_1	$\hat{x}_1$	(28)	-1,2	(13)	(28)·(13)
x_2	$\hat{x}_2$	(28)	-1,2	(14)	(28)·(14)
y_1	$\hat{y}_1$	(29)	-1,2	(15)	(29)·(15)
y_2	$\hat{y}_2$	(29)	-1,2	(16)	(29)·(16)
α_1	$\hat{\alpha}_1$	(30)	-1,2	(17)	(30)·(17)
		(31)	-1,2		(31)·(17)
α_2	$\hat{\alpha}_2$	(30)	-1,2	(18)	(30)·(18)
		(31)	-1,2		(31)·(18)
Measurand	**Measurand value**	**Combined standard uncertainty**	**Measurand kurtosis**	**Coverage factor**	**Expanded uncertainty**
x	(10)	(12)	(35)	(34)	(32)

Table 2 – Uncertainty budget for the measurement of the y-coordinate

Input quantities	**Values of input quantities**	**Standard uncertainties of input quantities**	**Kurtosis of input quantities**	**Sensitivity coefficients**	**Uncertainty contributions**
x_1	$\hat{x}_1$	(28)	-1,2	(20)	(28)·(20)
x_2	$\hat{x}_2$	(28)	-1,2	(21)	(28)·(21)
y_1	$\hat{y}_1$	(29)	-1,2	(22)	(29)·(22)
y_2	$\hat{y}_2$	(29)	-1,2	(23)	(29)·(23)
α_1	$\hat{\alpha}_1$	(30)	-1,2	(24)	(30)·(24)
		(31)	-1,2		(31)·(25)
α_2	$\hat{\alpha}_2$	(30)	-1,2	(25)	(30)·(26)
		(31)	-1,2		(31)·(27)
Measurand	**Measurand value**	**Combined standard uncertainty**	**Measurand kurtosis**	**Coverage factor**	**Expanded uncertainty**
y	(11)	(19)	(36)	(34)	(33)

4. Conclusions

1. The advantage of the goniometric method for determining the coordinates of an object on a plane is the absence of radiation from basic stations (passive location). Its implementation requires the presence of at least two basic stations equipped with goniometers.

2. The proposed scheme for implementing the measurement of the coordinates of an object on a plane using the goniometric method made it possible to obtain mathematical models for measuring the Cartesian coordinates of an object.

3. For the obtained mathematical models, based on the law of uncertainty propagation, expressions for standard uncertainties of measuring the coordinates of an object using the goniometric method were written, and calculations of sensitivity coefficients were made.

4. It is shown that for evaluation the expanded uncertainty of measuring the coordinates of an object using the goniometric method, it is advisable to use the excess method.

5. The uncertainty budgets of measuring the rectangular coordinates of an object are given, which can serve as a basis for automating the calculation of the uncertainty of measuring the coordinates of an object on a plane.

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Оцінювання невизначеності вимірювань координат об’єкту на площині гоніометричним методом

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Анотація

Обговорено особливості оцінки невизначеності вимірювання координат об'єкта на площині гоніометричним методом (тета-фіксація положення). Представлені вимірювальні моделі, які зв'язують координати об'єктів у локальній прямокутній системі координат з кутами, знайденими за допомогою гоніометрів. Модель включає поправки для визначення розташування базових станцій, а також поправки, пов'язані з неточностями в розташуванні станцій на північ. Наведено бюджети невизначеностей вимірювань прямокутних координат, які можуть бути основою для створення програмного забезпечення для автоматизації розрахунку невизначеностей вимірювань. Оцінки розширених невизначеностей знайдено методом ексцесу.

Ключові слова: координатна метрологія; гоніометричний метод; невизначеність вимірювань, бюджет невизначеності, метод ексцесів.