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MEASUREMENT UNCERTAINTY EVALUATION OF THE OBJECT PLANIMETRIC COORDINATES USING THE RHO-THETA METHOD

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Abstract

Determining the planimetric coordinates of an object is a topical task of coordinate metrology, which finds application in cartography, geodesy, location and navigation. The article considers the features of measurement uncertainty evaluation the coordinates of an object on a plane using the rho-theta method. The measurement model is substantiated. Expressions for estimating the numerical values of coordinates and their combined standard uncertainties are obtained. Expressions for estimating expanded uncertainties using the kurtosis method are found. The uncertainty budgets for measuring the planimetric Cartesian object coordinates are presented. Expressions for relative standard uncertainties in measuring coordinates are written and an example of their evaluation for real metrological characteristics of measuring instruments of a base station is considered.

Keywords: coordinate measurement; rho-theta fixing; measurement uncertainties evaluation, kurtosis method.

1. Introduction

In cartography [1], geodesy [2], radio navigation [3], radar [4], optical [5] and acoustic [6] locations the problem of determining the coordinates of a desired point (object) on a plane is widely used. Depending on the measuring capabilities of reference stations (BS), this issue is resolved by several methods: rangefinding (rho-rho) [7], goniometric (theta-theta) [8], and rangefinder-goniometric (rho-theta) [9].

The rho-theta method of measuring the planimetric coordinates of an object considered in this article is requires only a rangefinder and a goniometer installed on one base station (BS).

This refers to active location methods, since radiation from the BS is required to determine the range to the object.

2. Substantiation of the measurement model

The implementation scheme of measuring the planimetric coordinates of an object by the rho-theta measurement method is shown in Fig. 1.

The *OX* and *OY* axes form a right Cartesian coordinate system, with the *OY* axis directed to the north. The diagram shows a BS located at point O_{BS} with coordinates (x, y). Point *P* has the measurable coordinates (x_P, y_P).

The distance between the points O_{BS} and *P* ($O_{\mathrm{BS}}P$ = ρ) is measured using the rangefinder installed on the BS, and using the goniometer, the azimuth α is counted from the direction to the north to the direction to the object *P*.

Using the parameters ρ and α, the coordinates of the object *P* (x_P, y_P) can be determined using the expressions [2]:

$$x_P = x + \rho\sin(\alpha + \delta_N)\,; \qquad (1)$$

$$y_P = y + \cos(\alpha + \delta_N)\,, \qquad (2)$$

in which δ_N correction that takes into account the error of the basic station's reference to the north direction.

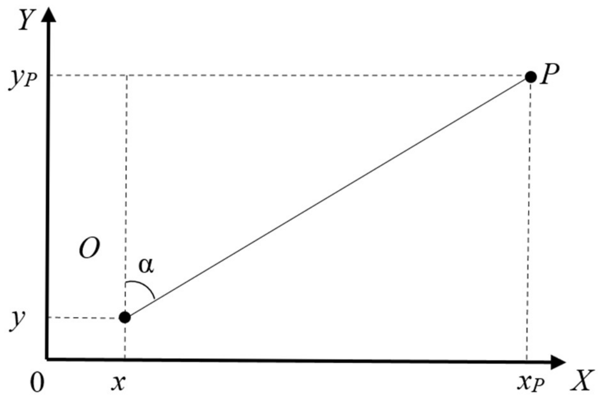


Fig. 1. Diagram of implementation planimetric coordinates measurement by the rho-theta method

3. Evaluation of the measurands and its measurement uncertainty

If the mathematical expectation of the correction $\hat{\delta}_N$ is taken to be equal to zero, then the values of the measured quantities (x_P, y_P) can be estimated from equations (1)-(2) by take the place into them of the values $\hat{x}$, $\hat{y}$, $\hat{\rho}$, $\hat{\alpha}$ of the input quantities:

$$\hat{x}_P = \hat{x} + \hat{\rho}\sin(\hat{\alpha})\,; \qquad (3)$$

$$\hat{y}_P = \hat{y} + \hat{\rho}\cos(\hat{\alpha})\,. \qquad (4)$$

The expression for the combined standard uncertainty (SU) of coordinate measurement can be written, in accordance with the law of propagation of uncertainty [10], as:

$$u_c(\hat{x}_P)=\sqrt{u^2(\hat{x})+c_\rho^2(x_P)u^2(\hat{\rho})+c_\alpha^2(x_P)[u^2(\hat{\alpha})+u^2(\hat{\delta}_N)]}\ , \quad (5)$$

where $u(\hat{x})$, $u(\hat{\rho})$, $u(\hat{\alpha})$ and $u(\hat{\delta}_N)$ are the standard uncertainties of the corresponding input quantities; $c_\rho(x_P)$ and $c_\alpha(x_P)$ are the corresponding sensitivity coefficients, which can be obtained as:

$$c_\rho(x_P)=\frac{\partial \hat{x}_P}{\partial \hat{\rho}}=\sin\hat{\alpha}\ . \quad (6)$$

$$c_\alpha(x_P)=\frac{\partial \hat{x}_P}{\partial \hat{\alpha}}=\hat{\rho}\cos\hat{\alpha}\ . \quad (7)$$

Equally, it can be expressed the combined SU of coordinate y_P measurement:

$$u_c(\hat{y}_P)=\sqrt{u^2(\hat{y})+c_\rho^2(y_P)u^2(\hat{\rho})+c_\alpha^2(y_P)[u^2(\hat{\alpha})+u^2(\hat{\delta}_N)]}\ , \quad (8)$$

where $u(\hat{y})$ is a SU of coordinate $\hat{y}$ measurement; $c_\rho(y_P)$ and $c_\alpha(y_P)$ corresponding sensitivity coefficients, which are determined by the expressions:

$$c_\rho(y_P)=\frac{\partial \hat{y}_P}{\partial \hat{\rho}}=\cos\hat{\alpha}\ ; \quad (9)$$

$$c_\alpha(y_P)=\frac{\partial \hat{y}_P}{\partial \hat{\alpha}}=-\hat{\rho}\sin\hat{\alpha}\ . \quad (10)$$

The SU of measurement of the slant range ρ of an object P is determined from the conforming boundaries of the maximum permissible error (MPE) of the rangefinder $\pm\theta_\rho$ under the assumption of a rectangular probability density functions (PDF) of the measurement error ρ within these boundaries:

$$u(\hat{\rho})=\theta_\rho\big/\sqrt{3}\ . \quad (11).$$

SU in measuring the location of a BS $(\hat{x},\hat{y})$ are determined from the boundaries of their MPEs $\pm\theta_x$, $\pm\theta_y$, attributing a rectangular PDF of these errors within the boundaries:

$$u(\hat{x})=\theta_x\big/\sqrt{3}\ ; \quad (12);$$

$$u(\hat{y})=\theta_y\big/\sqrt{3}\ . \quad (13)$$

In expressions (5), (8) $u(\hat{\alpha})$ is the standard instrumental uncertainty of azimuth α measurement using a goniometer, which can be found through the boundaries of the MPE error of the goniometer $\pm\theta_\alpha$ attributing a rectangular PDF of the instrumental error within these boundaries as:

$$u(\hat{\alpha})=\theta_\alpha\big/\sqrt{3}\ . \quad (14)$$

If the boundaries of the MPE of reference to the direction to the north are taken to be equal to $\pm\theta_N$, then, attributing a rectangular PDF of the reference error within these boundaries, the SU associated with this error can be estimated as:

$$u(\hat{\delta}_N)=\theta_N\big/\sqrt{3}\ . \quad (15).$$

Since the SU of all input quantities were estimated according to type B and were assigned a rectangular PDF, the expanded uncertainties of coordinate (x_P,y_P) measurement are best found using the kurtosis method [8]:

$$U(\hat{x}_P)=k(\eta_x)u(\hat{x}_P)\ ; \quad (16)$$

$$U(\hat{y}_P)=k(\eta_y)u(\hat{y}_P)\ , \quad (17)$$

where the coverage factors for a confidence level of p are calculated using the expression:

$$k(\eta)=\begin{cases}0{,}1085\eta^3+0{,}1\eta+1{,}96, \text{ for } p=0{,}95;\\ 0{,}12\eta^3+0{,}1\eta+2{,}0, \text{ for } p=0{,}9545,\end{cases} \quad (18)$$

and the kurtosis of the PDFs for (x_P,y_P) are calculated as:

$$\eta_x=\frac{-1{,}2\left\{c_\rho^4(x_P)u^4(\hat{\rho})+c_\alpha^4(x_P)[u^4(\hat{\alpha})+u^4(\hat{\delta}_N)]\right\}}{u_c^4(\hat{x}_P)}\ ; \quad (19)$$

$$\eta_y=\frac{-1{,}2\left\{c_\rho^4(y_P)u^4(\hat{\rho})+c_\alpha^4(y_P)[u^4(\hat{\alpha})+u^4(\hat{\delta}_N)]\right\}}{u_c^4(\hat{y}_P)}\ . \quad (20)$$

In this case, the uncertainty budgets of the object coordinate $(\hat{x}_P,\hat{y}_P)$ measurements will have the form given in Tables 1,2.

Table 1 – Uncertainty budget for the measurement of the x_P-coordinate

Input quantities (IQs)	**Values of IQs**	**SU of IQs**	**Kurtosis of IQs**	**Sensitivity coefficients**	**Uncertainty contributions**
ρ	$\hat{\rho}$	$u(\hat{\rho})$ (11)	-1,2	$c_\rho(x_P)$ (6)	$c_\rho(x_P)u(\hat{\rho})$
x	$\hat{x}$	$u(\hat{x})$ (12)	-1,2	1	$u(\hat{\delta}_x)$
α	$\hat{\alpha}$	$u(\hat{\alpha})$ (14)	-1,2	$c_\alpha(x_P)$ (7)	$c_\alpha(x)u(\hat{\alpha})$
δ_N	0	$u(\hat{\delta}_N)$ (15)	-1,2	$c_\alpha(x_P)$ (7)	$c_\alpha(x)u(\hat{\delta}_N)$
Measurand	**Measurand value**	**Combined SU**	**Measurand kurtosis**	**Coverage factor**	**Expanded uncertainty**
x_P	$\hat{x}_P$ (3)	$u_c(\hat{x}_P)$ (7)	η_x (19)	$k(\eta_x)$ (18)	U (16)

Table 2 – Uncertainty budget for the measurement of the y_P-coordinate

IQs	Values of IQs	SU of IQs	Kurtosis of IQs	Sensitivity coefficients	Uncertainty contributions
ρ	$\hat{\rho}$	$u(\hat{\rho})$ (11)	-1,2	$c_\rho(y_P)$ (9)	$c_\rho(y_P)u(\hat{\rho})$
y	$\hat{y}$	$u(\hat{y})$ (13)	-1,2	1	$u(\hat{y})$
α	$\hat{\alpha}$	$u(\hat{\alpha})$ (14)	-1,2	$c_\alpha(y_P)$ (10)	$c_\alpha(y_P)u(\hat{\alpha})$
δ_N	0	$u(\hat{\delta}_N)$ (15)	-1,2	$c_\alpha(y_P)$ (10)	$c_\alpha(y_P)u(\hat{\delta}_N)$
Measurand	**Measurand value**	**Combined SU**	**Measurand kurtosis**	**Coverage factor**	**Expanded uncertainty**
y_P	$\hat{y}_P$ (4)	$u_c(\hat{y}_P)$ (8)	η_y (20)	$k(\eta)$ (18)	U (17)

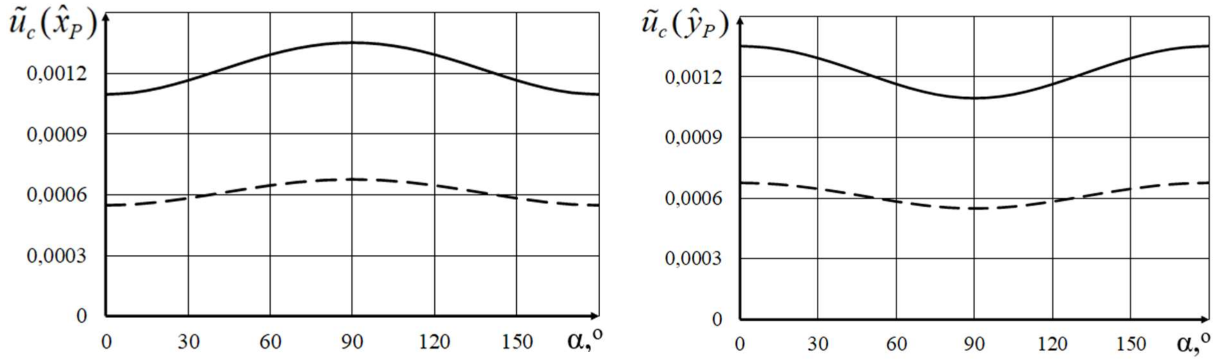
From expressions (5), (8) one can write expressions for the relative standard uncertainties of coordinate (x_P, y_P) measurement:

$$\tilde{u}(\hat{x}_P)=\frac{u(\hat{x}_P)}{\hat{\rho}}=\{\tilde{u}^2(\hat{x})+\sin^2(\alpha)\tilde{u}^2(\hat{\rho})+ +\cos^2(\hat{\alpha})[u^2(\hat{\alpha})+u^2(\hat{\delta}_N)]\}^{0,5}, \quad (21)$$

$$\tilde{u}_c(\hat{y}_P)=\frac{u(\hat{y}_P)}{\hat{\rho}}=\{\tilde{u}^2(\hat{y})+\cos^2(\hat{\alpha})\tilde{u}^2(\hat{\rho})+ +\sin^2(\hat{\alpha})[u^2(\hat{\alpha})+u^2(\hat{\delta}_N)]\}^{0,5}, \quad (22)$$

where $\tilde{u}(\hat{\delta}_x)=u(\hat{\delta}_x)/\hat{\rho}$; $\tilde{u}(\hat{\delta}_y)=u(\hat{\delta}_y)/\hat{\rho}$ are relative standard uncertainties of corrections $\hat{\delta}_x$, $\hat{\delta}_y$; $\tilde{u}(\hat{\rho})=u(\hat{\rho})/\hat{\rho}$ is relative SU of measurement ρ.

For the values $\hat{\rho}$=1000 m and $\hat{\rho}$=2000 m; θ_ρ=1.5 m; $\theta_x=\theta_y$=1.8 m; θ_α=0.6 mrad; θ_N=0.1 mrad; given in [11], the dependences $\tilde{u}(\hat{x}_P)$ and $\tilde{u}(\hat{y}_P)$ on α were obtained, shown in Fig. 2.



a) b)

Fig. 2. Dependences $\tilde{u}(\hat{x}_P)$ (a) and $\tilde{u}(\hat{y}_P)$ (b) on α for $\hat{\rho}$=1000 m (—) and $\hat{\rho}$=2000 m (– –)

4. Conclusions

1. The preference of the rho-theta measurement method for determining the planimetric coordinates of an object is the use of only one base station equipped with a range finder and a goniometer.

2. Models for measuring the abscissa and ordinate of the sought object using the range-angle method are obtained based on the constructed scheme for its implementation.

3. For the proposed measurement models, expressions for standard uncertainties in measuring the coordinates of an object by the rho-theta measurement method were written and the sensitivity coefficients included in them were calculated.

4. It was shown that it is reasonable to use the kurtosis method to estimate the expanded uncertainty in measuring the coordinates of an object by the rho-theta measurement method.

5. The uncertainty budgets for measuring the planimetric Cartesian object coordinates are presented, which can be used as the basis for a computer program facilitating the routine calculation of measurement uncertainty.

6. The dependences of relative uncertainties in measuring the object coordinates on the azimuth are calculated for real metrological characteristics of measuring instruments of a base station is given.

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Оцінювання невизначеності вимірювань координат об’єкту далекомірно-кутовим методом на площині

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Анотація

Визначення координат об'єкта на площині є актуальним завданням координатної метрології, яке знаходить своє застосування у картографії, геодезії, локації та навігації. У статті розглядаються особливості оцінювання невизначеності вимірювання координат об'єкта на площині кутомірно-далекомірним методом. Обгрунтовано модель вимірювань. Отримана процедура оцінювання невизначеності вимірювань абсциси і ординати об’єкту на площині, яка включає в себе оцінки числового значення координат, їх сумарні стандартні невизначеності та розширені невизначеності, які оцінюються методом ексцесів. Наводиться приклад оцінювання відносних стандартних невизначеностей вимірювання абсциси і ординати об’єкту для реальних метрологічних характеристик засобів вимірювальної техніки базової станції.

Ключові слова: вимірювання координат; далекомірно-кутомірний; оцінювання невизначеностей вимірювань, бюджет невизначеностей, метод ексцесів.

