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THEORETICAL AND PRACTICAL ASPECTS OF BIRGE RATIO METHOD FOR ADJUSTING UNCERTAINTY IN MULTIVARIATE MEASUREMENTS

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Abstract

In the paper, we present the multivariate location-scale model connected to the multivariate Birge ratio method, a new approach to model the dark uncertainty which is usually present when the results of individual studies are pooled together. In the empirical illustration, the approach is applied to the measurement results used to study the presence of the effectiveness of the hypertension treatment. The findings are compared to the ones obtained when the multivariate random effects model is used. Both models confirm that the hypertension treatments can lower systolic blood pressure and diastolic blood pressure as well as lead to considerable reduction of the risks of cardiovascular disease and stroke. Furthermore, the multivariate Birge ratio method produces more precise estimators of the overall mean vector by resulting in considerably smaller standard errors and narrower confidence intervals.

Keywords: Dark uncertainty; multivariate Birge ratio method; multivariate location-scale model; multivariate random effects model.

1. Introduction

Multivariate random effects model and multivariate location-scale model are the comonly used approaches for assessing and modeling dark uncertainty in multivariate measurements (see Jackson et al. (2010), Gasparrini et al. (2012), Jackson et al. (2020), Bodnar and Bodnar (2024b), Bodnar and Bodnar (2025)). The multivariate random effects model suggests an additive adjustment of the reported uncertainties, while the multivariate location-scale model is connected to the Birge ratio method and adjusts the reported uncertainties in the multiplicative way. We apply both models to the data which measure the effectiveness of the hypertension treatment and compare their ability to assess the dark uncertainty.

The multivariate random effects model generalizes the univariate approach which is mostly used to conduct meta-analysis in medicine, chemistry, and metrology (see, e.g., Hardy and Thompson (1996), Rukhin (2013), Bodnar et al. (2016), Turner et al. (2015), Bodnar et al. (2017), Guolo and Varin (2017), Veroniki et al. (2019)). The multivariate location-scale model together with the Birge ratio method as developed in Bodnar and Bodnar (2025) extends the univariate location-scale model and it is preferable in metrology and physics (see Birge (1932), Bodnar and Elster (2014), Weise and Wöger (2000), Tiesinga et al. (2021), Bodnar and Eriksson (2023)). Recently, using Bayesian model selection, Bodnar and Eriksson (2023) conclude that the Birge ratio method outperforms the random effects models based on data used in the determination of physical constants.

In this paper, we contribute to the existent literature by comparing the multivariate location-scale model connected to the Birge ratio method and the multivariate random effects model. In the comparison study, we analyse the ability of both models to estimate the overall mean vector for data used to study the effectiveness of the hypertension treatment. Both approaches find that the conducted hypertension treatment lowers both systolic blood pressure and the diastolic blood pressure and considerably reduces risks of cardiovascular disease and stroke. Moreover, the application of the multivariate Birge ratio method to the considered data leads to more precise estimators of the overall mean vector by producing considerably smaller standard errors and narrower confidence intervals in comparison to the ones obtained when the multivariate random effects model is used.

The rest of the paper is organized as follows. The multivariate random effects model and the multivariate location-scale model together with the Birge ratio method are presented in Section 2, while they are applied to the data dealing with the effectiveness of the hypertension treatment in Section 3. Final remarks are provided in Section 4.

2. Methods for modeling multivariate dark uncertainty

We consider a study in which p features are measured simultaneously. Let $\mathbf{x}_i$ be the p-dimensional vector of the measurement results from the i-th study for $i=1,\dots,n$ which is reported together with the covariance matrix $\mathbf{U}_i$, the multivariate measure of uncertainty in the i-th observation vector.

The multivariate random effects model adjusts the reported multivariate uncertainties additively and it is defined by

$$\mathbf{x}_i = \boldsymbol{\mu} + \boldsymbol{\lambda}_i + \boldsymbol{\varepsilon}_i \quad \text{with } \boldsymbol{\lambda}_i \sim \mathrm{N}_p(\mathbf{0},\boldsymbol{\Psi}) \text{ and } \boldsymbol{\varepsilon}_i \sim \mathrm{N}_p(\mathbf{0},\mathbf{U}_i), \quad (1)$$

where $\{\boldsymbol{\lambda}_i\}_{i=1,...,n}$ and $\{\boldsymbol{\varepsilon}_i\}_{i=1,...,n}$ are assumed to be mutually independent. Moreover, $\boldsymbol{\lambda}_i$ and $\boldsymbol{\varepsilon}_i$ are assumed to be normally distributed. The parameter vector $\boldsymbol{\mu}$ is the overall mean vector, the main parameter of the model, while the matrix $\boldsymbol{\Psi}$ is the between-study covariance matrix, the nuisance parameter of the model (1) that is used to assess the multivariate dark uncertainty. Different methods are developed in the literature to estimate $\boldsymbol{\mu}$ and $\boldsymbol{\Psi}$. While Gasparrini et al. (2012) and Jackson et al. (2013) discuss the frequentist approaches, Bodnar and Bodnar (2023), Bodnar and Bodnar (2024a), and Bodnar and Bodnar (2024b) develop Bayesian methods, recently.

Another method to adjust multivariate measurements is based on the multivariate location-scale model, a generalization of the univariate approach, expressed as

$$\mathbf{x}_i = \boldsymbol{\mu} + \mathbf{B}\boldsymbol{\varepsilon}_i \quad \text{with} \quad \boldsymbol{\varepsilon}_i \sim \mathrm{N}_p(\mathbf{0},\mathbf{U}_i), \tag{2}$$

where the measurement errors $\boldsymbol{\varepsilon}_1,...,\boldsymbol{\varepsilon}_n$ are assumed to be mutually independent. The parameter vector $\boldsymbol{\mu}$ is the overall mean vector and it usually presents the main parameter of the model, similarly to the multivariate random effects model. The matrix $\mathbf{B}$ is a diagonal matrix with entries $b_1,...,b_p$. This matrix is used to adjust the reported unceratinties from the individual studies in the multiplicative way. If $p = 1$, the multivariate location-scale model becomes the univariate location-scale model and the multivariate Birge ratio method connected to the multivariate location-scale model becomes the univariate Birge ratio method, previously studied in Birge (1932), Bodnar and Elster (2014), Bodnar and Eriksson (2023), among others.

The multivariate Birge ratio model has recently been developed in Bodnar and Bodnar (2025), who derive the maximum likelihood estimator for the parameters $\boldsymbol{\mu}$ and $\mathbf{B}$ (or $\mathbf{b}$) of model (2). Let $\mathbf{b} = (b_1,...,b_p)^\top$ be the vector of the diagonal elements of the matrix $\mathbf{B}$ and define $\tilde{\boldsymbol{b}} = \left(\boldsymbol{b}_1^{-1},\ldots,\boldsymbol{b}_p^{-1}\right)^\top$ as the vector which consist of the inverses of the diagonal elements of $\mathbf{B}$. Then, we get

(i) The maximum likelihood estimator for $\mathbf{B}$ is given by

$$\widehat{\boldsymbol{B}} = diag(\hat{\tilde{\boldsymbol{b}}}^{-1}), \tag{3}$$

where $\mathbf{b}$ is a vector with positive elements that solves the following system of quadratic equations

$$\tilde{\boldsymbol{b}} \odot (\boldsymbol{Q}\tilde{\boldsymbol{b}}) = \mathbf{1}_{\mathrm{p}} \tag{4}$$

with

$$\boldsymbol{Q} = \frac{\mathbf{1}}{\boldsymbol{n}}\sum_{i=1}^{n}\boldsymbol{A}_i\boldsymbol{U}_i^{-1}\boldsymbol{A}_i - \frac{\mathbf{1}}{\boldsymbol{n}}\sum_{i=1}^{n}\boldsymbol{A}_i\boldsymbol{U}_i^{-1}\left(\frac{\mathbf{1}}{\boldsymbol{n}}\sum_{i=1}^{n}\boldsymbol{U}_i^{-1}\right)^{-1}\frac{\mathbf{1}}{\boldsymbol{n}}\sum_{i=1}^{n}\boldsymbol{U}_i^{-1}\boldsymbol{A}_i,$$

where $\mathbf{A}_i = diag(\mathbf{x}_i)$, i.e., a diagonal matrix with diagonal elements equal to the corresponding elements of the vector $\mathbf{x}_i$ and $\mathbf{1}_p$ is the p-dimensional vector of ones. The symbol $\odot$ denotes the Hadamard product.

(ii) The maximum likelihood estimator for μ is given by

$$\hat{\boldsymbol{\mu}} = \bar{x}(\widehat{\boldsymbol{B}}) = \widehat{\boldsymbol{B}}\left(\sum_{i=1}^{n}\boldsymbol{U}_i^{-1}\right)^{-1}\sum_{i=1}^{n}\boldsymbol{U}_i^{-1}\widehat{\boldsymbol{B}}^{-1}\boldsymbol{x}_i\,. \tag{5}$$

Moreover, using the generalized likelihood ratio approach, Bodnar and Bodnar (2025) derive asymptotic marginal confidence intervals for each component of the overall mean vector $\boldsymbol{\mu}$ which for the j-th component is given by

$$\mathcal{C}(\mu_j) = \left[\hat{\mu}_j - u_{(1+\gamma)/2}\sqrt{d_{jj}}, \hat{\mu}_j + u_{(1+\gamma)/2}\sqrt{d_{jj}}\right], \tag{6}$$

where u_β is the β-quantile of the standard normal distribution, $\hat{\mu}_j$ is the j-th element of $\hat{\boldsymbol{\mu}}$, and d_{jj} is the j-th diagonal element of $\widehat{\boldsymbol{B}}\left(\sum_{i=1}^{n}\boldsymbol{U}_i^{-1}\right)^{-1}\widehat{\boldsymbol{B}}$.

It is noted that the system of quadratic equations (4) cannot be solved analytically in general. One of possible methods to find the solution of (4) is the usage of the Buchberger algorithm which is based on the Gröbner basis and is implemented in Mathematica (see Buchberger and Winkler (1998), Buchberger (2001), Cox et al. (2015)). However, when $p = 2$, the system of quadratic equations (4) can be rewritten as

$$\begin{cases} q_{11}\tilde{b}_1^2 + q_{12}\tilde{b}_1\tilde{b}_2 = 1, \\ q_{21}\tilde{b}_1\tilde{b}_2 + q_{22}\tilde{b}_2^2 = 1, \end{cases} \tag{7}$$

where q_{jm} are the elements of $\mathbf{Q}$ for $j,m = \{1,2\}$ with $q_{12} = q_{21}$. Then, the unique solution of (7) consisting of positive elements in $\tilde{\boldsymbol{b}}$, is given by

$$\hat{\tilde{b}}_1 = \frac{1}{\sqrt{q_{11}}\sqrt{1+\frac{q_{12}}{\sqrt{q_{11}q_{22}}}}}\,, \hat{\tilde{b}}_2 = \frac{1}{\sqrt{q_{22}}\sqrt{1+\frac{q_{12}}{\sqrt{q_{11}q_{22}}}}}. \tag{8}$$

Another important special case corresponds to diagonal covariance matrices $\mathbf{U}_i$. In this case, the matrix $\mathbf{Q} = (q_{jm})_{j,m=\{1,...,p\}}$ is also diagonal with the diagonal elements given by

$$q_{jj} = \frac{1}{n}\sum_{i=1}^{n}\frac{x_{i,j}^2}{u_{i,jj}} - \frac{\left(\frac{1}{n}\sum_{i=1}^{n}x_{i,j}/u_{i,jj}\right)^2}{\frac{1}{n}\sum_{i=1}^{n}1/u_{i,jj}},$$

where $\mathbf{x}_i = (x_{i,1},...,x_{i,p})^\top$ and $\mathbf{U}_i = (u_{i,jm})_{j,m=\{1,...,p\}}$. Hence, the inverse of j-th element of $\mathbf{b}$ becomes

$$\hat{b}_j = \sqrt{q_{jj}}$$

and it coincides with the Birge ratio computed for the j-the measured feature based on a univariate sample of measurement results for this feature (see Weise and Wöger (2000)). Also, the maximum likelihood estimator of the j-th component of $\boldsymbol{\mu}$ becomes

$$\hat{\mu}_j = \frac{\sum_{i=1}^{n}x_{i,j}/u_{i,jj}}{\sum_{i=1}^{n}1/u_{i,jj}},$$

which is the weighted mean estimator, widely spread in the univariate case (see Bodnar and Elster (2014), Strawderman and Rukhin (2010)).

3. Assessing dark uncertainty in measurements for the effectiveness of the hypertension treatment

In this section, we apply the multivariate Birge ratio method to assess the dark uncertainty in real data which consist of the results of ten studies on the effectiveness of the hypertension treatment. In each individual study, four features were measured which are systolic blood pressure (SBP), diastolic blood pressure (DBP), cardiovascular disease (CVD), and stroke. We refer to Wang et al. (2005) for the detailed description of the considered data. Additionally, three approaches from frequentist statistics are used to infer the parameters of the multivariate random effects model.

The measurements of each study together with reported covariance matrices are available Table 1 of Riley et al. (2015). Some of the provided covariance matrices however appear not to be positive definite. These covariance matrices are adjusted by adding a diagonal matrix with the same diagonal elements to ensure that all reported covariance matrices are positive definite. The aim of the study was to analyze whether hypertension treatments can lower SBP and DBP and whether these treatments lead to the significant reduction of the risks of CVD and stroke. The variables SBP and DBP are defined as differences between the mean results in the group who received treatments and in the placebo group (without treatment), while the variables CVD and stroke are defined as the logarithms of the hazard ratio computed for the two groups. As such, negative values indicate the presence of the treatment effect. Also, the estimators of the overall mean vector together with the uncertainties obtained by the multivariate Birge ratio method and the multivariate random effects model will be compared in this section.

The maximum likelihood estimates for the parameters of the multivariate location-scale model (2) are computed as

$$\hat{\boldsymbol{\mu}} = \begin{pmatrix} -9.584 \\ -3.682 \\ -0.112 \\ -0.400 \end{pmatrix}, \hat{\boldsymbol{B}} = \begin{pmatrix} 2.719 & 0 & 0 & 0 \\ 0 & 3.302 & 0 & 0 \\ 0 & 0 & 0.683 & 0 \\ 0 & 0 & 0 & 0.955 \end{pmatrix}.$$

It is interesting that two diagonal elements of the matrix $\hat{\boldsymbol{B}}$ are larger than one, while the other two values are smaller than one, indicating that the reported uncertainties in the case of the SBP and DBP measurements are too small, and they need to be corrected. This is not the case with the uncertainties reported in the case of the CVD and stroke variables.

In Table 1, the estimates of the overall mean vector $\boldsymbol{\mu}$ are depicted together with their standard errors and 95% confidence intervals computed for each component of $\boldsymbol{\mu}$ separately.

Table 1: Estimates, standard errors, and 95% confidence intervals for the components of the overall mean vector **μ** obtained by applying the multivariate Birge ratio method derived in Bodnar and Bodnar (2025), the maximum likelihood and the restrictive maximum likelihood approaches under the multivariate random effects model described in Gasparrini et al. (2012), and the method of moments under the multivariate random effects model from Jackson et al. (2013).

	μ_1 (SBP)	μ_2 (DBP)	μ_3 (CVD)	μ_4 (stroke)
MLE, Multivariate Birge ratio method				
estimate	-8.79	-4.001	-0.226	-0.388
stand. error	0.506	0.357	0.046	0.073
conf. inter.	[-9.783, -7.801]	[-4.701, -3.300]	[-0.316, -0.136]	[-0.531, -0.244]
MLE, Gasparrini et al. (2012)				
estimate	-10.177	-4.622	-0.232	-0.323
stand. error	0.867	0.497	0.071	0.093
conf. inter.	[-11.875,-8.478]	[-5.596,-3.647]	[-0.371,-0.093]	[-0.505,-0.140]
REML, Gasparrini et al. (2012)				
estimate	-10.224	-4.646	-0.233	-0.321
stand. error	0.927	0.530	0.071	0.095
conf. inter.	[-12.039,-8.408]	[-5.685,-3.608]	[-0.372,-0.093]	[-0.507,-0.134]
Method of moments, Jackson et al. (2013)				
estimate	-9.923	-4.500	-0.228	-0.335
stand. error	0.634	0.394	0.072	0.107
conf. inter.	[-11.166,-8.681]	[-5.267,-3.724]	[-0.369,-0.086]	[-0.545,-0.125]

While no large differences are present in the estimators of the corresponding elements of the mean vector, the standard uncertainties differ significantly.

The application of the multivariate Birge ratio method results in the smallest standard errors and narrowest confidence intervals. This approach is followed by the multivariate random effect models with the parameters estimated by the method of moments. Finally, the widest confidence intervals are obtained when the parameters of the multivariate random effects model are fitted by the restrictive maximum likelihood method.

Independently of the employed model to assess the presence of the dark uncertainty in the considered multivariate measurements and the method used to estimate the parameters in the case of the multivariate random effects model, all results in Table 1 confirm the presence of the effectiveness of the hypertension treatments in all four variables. These findings are in line with the previous results reported in Riley et al. (2015) who draw the same conclusion by fitting the multivariate random effects model to the data.

4. Summary

Combining the measurements of individual studies and the results of the interlaboratory studies into a single consensus value is an important topic of modern research with applications in medicine, chemistry, physics, metrology, among others. The situation becomes even more challenging when several features are measured in each individual study and it is reported together with the covariance matrix, the multivariate measure of uncertainty.

In this study, we discuss the novel procedure to pool multivariate results of individual studies into the consensus vector by employing the multivariate location-scale model in connection with the multivariate Birge ratio methods, recently developed in Bodnar and Bodnar (2025). The estimators of the model parameters are obtained by applying the maximum likelihood approach. The new procedure is compared to the multivariate random effects model via an empirical application dealing with the study of the effectiveness of the hypertension treatments. It is found that the application of the multivariate Birge ratio method results in a more precise estimator of the overall mean vector. The computed standard uncertainties of the components of the overall mean vector are considerable smaller than the uncertainties obtained when the multivariate random effects model is used with different estimation procedures. Furthermore, independently of the employed model, the presence of the effectiveness of the hypertension treatments is found.

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Теоретичні та практичні аспекти методу коригування невизначеності багатовимірних вимірювань за допомогою коефіцієнта Бірге

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Анотація

У статті ми представляємо багатовимірну модель масштабу розташування, пов'язану з багатовимірним методом коефіцієнта Бірге, новим підходом до моделювання темної невизначеності, яка зазвичай присутня, коли результати окремих досліджень об'єднуються. В емпіричній ілюстрації підхід застосовується до результатів вимірювань, що використовуються для вивчення ефективності лікування гіпертензії. Результати порівнюються з результатами, отриманими при використанні багатовимірної моделі випадкових ефектів. Обидві моделі підтверджують, що лікування гіпертензії може знизити систолічний та діастолічний артеріальний тиск, а також призвести до значного зниження ризику серцево-судинних захворювань та інсульту. Крім того, багатовимірний метод коефіцієнта Бірге дає точніші оцінки загального середнього вектора, що призводить до значно менших стандартних помилок та вужчих довірчих інтервалів.

Ключові слова: темна невизначеність; багатовимірний метод коефіцієнта Бірге; багатовимірна модель масштабу місця; багатовимірна модель випадкових ефектів.

