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STATISTICAL ASSESSMENT OF THE RELIABILITY OF DECISIONS ON THE STATE OF CONTROLLED TECHNOLOGICAL PROCESS BASED ON E. VOLODARSKYI* APPROACH

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Abstract. The article concerns the issues of using control charts to study the parameters describing the state of the products in their production process. The identification of reliability of a decision is based on the assessment of disturbances occurring in it is discussed. Using is the method proposed by Yevhen Volodarskyi, that was based on Bayesian approach. The influence of measurement errors and their distribution of probability on the correctness of the decisions taken is considered. In the article two estimations of conformity of technological process to the norms based on the results of its control are considered. The first assessment is a-priori probability or reliability of the control result, which is performed before the control procedure and is based on a-priori data about the process and measurement error. The paper proposes the use of the second assessment, namely the posterior probability of compliance of the technological process with the norms. This assessment of compliance is performed after the control result is obtained, when only half of the set of elementary events contributing to the occurrence of one of the control results is left for evaluation. The use of this estimation allows doubling the statistical reliability of the control result estimation. The effectiveness of assessing the compliance of the technological process with the standards established for the uniform distribution of the values of its controlled parameters and their measurement errors is also determined**.**

Keywords: statistical process control, process quality control, measurement errors, statistical reliability of decisions, conformity assessment, probability of Bayes, a-priori and a-posteriori probability, Python data modeling.

1. Introduction

Product quality is determined by the conformity of all technical parameters of the technological process to predetermined norms and standards. In addition, to ensure product quality, it is necessary to control the conformity of manufactured products with the requirements of these norms and standards. In production, this control is carried out by measuring the relevant characteristic parameters of the technological process and the manufactured products.

Randomness of destabilizing factors leads to dispersion of values of characteristic parameters of products in production processes. The randomness of the deviation of the parameters is based on the specific theoretical law of distribution of the possible results [9].

Conformity assessment is all about gaining initial information about a given process's current state through comparison of process characteristics or indicators with pre-defined requirements [9, 10]. Based on the functional purpose of the product, standards are established or calculated. Compliance testing is used to determine the actual state of the product, that is, whether it conforms to the standards [9, 11].

Quality assurance is based on creating quality through the production process, not by inspecting its results. The detected non-conformity is an event that has already occurred and cannot be prevented, so it is always too late to inspect. However, the occurrence of nonconformities can be prevented by proactive management of process characteristics [9].

Assessment of compliance of technological processes or manufactured products with the norms is very often performed with the help of control charts [12]. The use of the tool of control charts for conformity assessment implies the absence of errors in the

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* Doctor of Technical Sciences Yevhen T. Volodarskyi, Professor of the National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnic Institute” in Chair of Information and Measuring Technologies, died suddenly on February 9, 2025, in Kyiv at the age of 80 in the fullness of his creative power.

Prof. Ye. Volodarskyi awarded by the President of Ukraine “Honored Lecturer of Ukraine”, was also President of the Academy of Metrology of Ukraine association, Chairman of the sub-commission “Metrology” at the State Accreditation Agency of Ukraine, senior member of IEEE, and member of the international scientific council of the Polish journal PAR (Measurement Automation Robotics.

During all his professional live he initiated, work himself, and managing the new research problems in the Measurement Science up to the last moments of his hard-working life. He used the current statistical approach to errors and uncertainties of measurements according to convention of guide GUM [1] and always based on the newest world literature. This activity covers accuracy problems of the technical inspection, and the development of methods for assessing the accuracy and reliability of the continuous production process control including measurement systems based on different types of control cards.

Prof. E. Volodarski is author of several books and over 300 publications. As an example, during last 15 years only, together with metrologists from Ukraine and Poland, including authors he published over 60 international publications, mainly in English and Polish. Some last works [2-4], [5-9] are given in the literature of this paper.

Prof. E. Volodarskyi also initiated and participated in the formulation and development of this work.

measurement results of characteristic parameters of the technological process/product or the absence of significant influence of measurement error. In fact, the uncertainty of measurement results is present in real measurements and is often not considered when controlling production using control charts.

Ignoring the uncertainty of the measurement result can lead to the fact that the qualitative assessment of the state of the technological process/product may be false and not correspond to the true state. Uncertainty of the measurement result leads to uncertainty of decision making about process/product compliance. This paper is concerned with the study of a method to improve the statistical reliability of the estimation of conformity of the technological process to norms. The approach considered was proposed by Prof. Ye. T. Volodarskyi.

A priori probabilities based on facts that have been confirmed over time are used as the initial information for assessing compliance. Based on established knowledge about the problem being modeled, the facts can be evaluated. The initial information in the design of information and measurement systems is the a-priori probability of conformity assessment. These are related to characteristics of technological procedure. In order to obtain data on the progress of the process, a measurement procedure is used that is known to be error prone. The state of the technological process is adequately reflected by the results of control and measurement operations if the measurement errors are insignificant. Erroneous decisions are absent in this case [9].

Issues related to the influence of systematic additive measurement error, when it does not occur in the production process, are considered in the monograph [12]. It is assumed that the presence of a constant error can be considered in the calibration of control charts when determining the so-called empirical control limits. An additive error in the law of distribution shifts the control values of the technological process. Erroneous decisions may occur regarding the process [9].

The article [13] examines the criteria for identifying measurement errors during process control. Identifying control points and their locations allows for identifying measurement error impact at the start of process disturbances. The subsequent phase in enhancing the reliability of process control entails the consideration of the impact not only of the additive component of measurement errors but also of the multiplicative component during the calibration of control charts [9].

A study [2-4] analyzed the probability and nature of erroneous decisions with a real characteristic of measurement transformation and considered also methods to increase the reliability of decisions.

The likelihood of incorrect decisions based is on the assumption that one event may happen, using the Bayes approach to reduce the number of potential causes by twofold. This increases the reliability of decisions [14, 15].

When taking into consideration the outcomes that are tainted by bias, the corrections applied to decision errors, the reproduction of the general population, and the determination of the probability of a true positive result, the primary advantages of the Bayesian approach become evident [16]. The Bayesian approach estimates event probabilities based on experimental data. It uses prior knowledge to calculate probabilities of events [9].

2. Process quality control

In this paper we will consider the case when the compliance of the technological process with the norms is determined by one parameter x. This parameter is characterized by the nominal value x_{nom}, which ideally should reproduce the parameter x and which is defined by the standards for production.

During production, the controlled parameter's value differs from its nominal value, varying across objects due to production errors and unstable factors. These errors result in random values, making compliance difficult [17]. Limits x_l and x_u are placed on these values. A conformity assessment procedure is performed to determine if objects comply with the standard [18].

The paper assumes that the random variable of the process parameter x has a uniform distribution law. The permissible values of the characteristic parameter x are defined by the production standards. Finding the value of the parameter x in the range of permissible values indicates that the technological process is normal and complies with the standards for production:

$$x_l \leq x \leq x_u \quad (1)$$

where x_l: lower boundary of the range of permissible values of the characteristic parameter of the technological process; x_u: upper boundary of the range of permissible values.

The paper considers the case when the boundaries of the range of permissible values (1) are symmetric with respect to the nominal value $x_{nom} = \frac{x_u - x_l}{2}$. In addition to the permissible values of the parameter x, which are allowed by the norms in the production, the value of the parameter x can go beyond the permissible range (1) and accordingly take values inadmissible by the standard. Let's assume that all possible values of the characteristic parameter of the technological process x belong to the range of possible values with lower boundary x_{min} and upper boundary x_{max}, respectively:

$$x_{min} \leq x \leq x_{max}. \quad (2)$$

In addition, we assume that the limits of possible values (2) of the parameter x are also placed symmetrically with respect to the nominal value x_{nom}. Let's introduce a value which characterizes the length of the interval of permissible values $N = x_u - x_l$. Also, let us introduce $H = x_{nom} - x_{min} = x_{max} - x_{nom}$, which characterizes the half-range of the range of possible values of the parameter x. Fig. 1 shows the graphical representation of the entered values and the corresponding permissible and possible range of values of the parameter x.

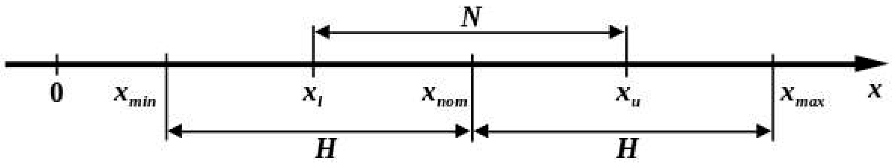

Fig. 1. Ranges of values of the characteristic parameter x of the technological process

Process control involves measuring a characteristic process parameter x and a measurement result z:

$$z = z(x). \tag{3}$$

The result z of the characteristic parameter x measurement is used to construct control charts.

As indicated in the extant literature, including the works of experts such as [19], control charts have been employed for a considerable period to evaluate the conformance of technological processes to specified standards. The nominal value of the characteristic parameter is assumed when constructing the control charts, provided that the technological process is in a statistically controlled state [18]. As illustrated in Figure 2, this nominal value corresponds to the center line (CL) on the graph. The nominal value undergoes either reproduction or calculation. The upper UCL and lower LCL limits are relative to CL (see Figure 2). These lines are referred to as "action lines." These lines represent the limits of the tolerance interval, i.e., the permissible deviation and the characteristic parameter from the nominal value. The tech process conforms to standards (1) [20] because its parameter is in the tolerance interval.

Fig. 2 shows a control chart on which the measured values z_i (3) of a characteristic process parameter x are plotted.

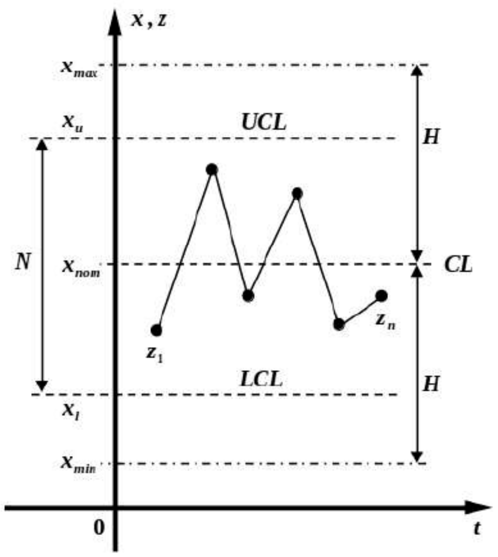

Fig. 2. Process control chart

On the other hand, over time the aging of the production element base can lead to non-compliance of the technological process with the norms, but in this paper, we will be interested in the event of the process fault occurrence at some point in time and the appearance of the corresponding signal from the control chart.

For the sake of simplicity of mathematical calculations, in this paper it was decided not to consider that control charts are used for random processes distributed beyond normal law.

3. Assessment of the result of process control

Let's consider the influence of measurement error on the measurement result on the basis of which decisions about the state of the technological process are made. In this paper we will consider additive random measurement error y. Then equation (3) of the measurement result z of the parameter x will have the form:

$$z = x + y. \tag{4}$$

3.1. No measurement uncertainty

Assuming that there is no measurement error $y = 0$, then the measurement result (4) will correspond exactly to the value of the characteristic process parameter:

$$z = x. \tag{5}$$

As a result of the conducted control, the researcher decides about the state of the technological process. As already mentioned for this purpose he uses control charts, which are built on the measurement data of characteristic parameters of the technological process. In the ideal case of no measurement error, the measurement result corresponds to the value of the technological process parameter and the decision made corresponds to the actual state of the technological process. In this case, there are two possible outcomes of control:

- event A: the technological process is normal and complies with standards,
- event $\overline{A}$: the technological process is faulty and does not meet the standards.

These outcomes constitute a complete group of mutually exclusive events:

$$\Omega_1 = \{A, \overline{A}\}. \tag{6}$$

And accordingly, the probabilities of these events (6) satisfy the expression:

$$P(A) + P(\overline{A}) = 1. \tag{7}$$

The event A will occur if condition (1) is met, i.e. the measurement result z of the process parameter is within the range of acceptable values. The event $\overline{A}$ will occur if condition (1) is not met.

The random nature of the parameter x change leads to the possibility of calculating the probability of occurrence of each of the possible events (6) when the expression (7) is fulfilled. These events (6) are the same both for the actual state of the process before the control and for the result of the conducted control, due to the absence of measurement error (5). The probabilities of occurrence of events (6) depend only on the values of the characteristic parameter x of the technological process and the boundaries of the intervals (1-2), i.e. on the state of the process itself.

The probabilities of occurrence of events (6) are favored by the size of the corresponding interval into which the parameter x value falls (Fig. 1). Therefore, the probabilities of occurrence of events (6) are proportional to the lengths of the corresponding ranges (Fig. 1). To calculate the probabilities (7) for the uniform law of distribution of the parameter x, it is necessary to determine the sizes of the corresponding ranges that favor events (6). The expression for probabilities (7) is given in Table 1.

Table 1. Probabilities (7) of the occurrence of control results (6)

A	$\overline{A}$
$P(A) = P(x_l \leq z \leq x_u)$	$P(\overline{A}) = P[(x_{min} \leq z < x_l) \cup (x_u < z \leq x_{max})]$
$P(A) = \int_{x_l}^{x_u} f_1(x)dx$	$P(\overline{A}) = \int_{x_{min}}^{x_l} f_1(x)dx + \int_{x_u}^{x_{max}} f_1(x)dx$
$P(A) = \int_{x_l}^{x_u} f_1(x)dx = \int_{x_l}^{x_u} \frac{1}{2H}dx = \frac{x_u - x_l}{2H} = \frac{N}{2H}$	$P(\overline{A}) = \int_{x_{min}}^{x_l} f_1(x)dx + \int_{x_u}^{x_{max}} f_1(x)dx =$ $= \int_{x_{min}}^{x_l} \frac{1}{2H}dx + \int_{x_u}^{x_{max}} \frac{1}{2H}dx =$ $= \frac{x_l - x_{min}}{2H} + \frac{x_{max} - x_u}{2H} = \frac{2H - N}{2H} = 1 - \frac{N}{2H}$
$P(A) = S_A = hN = \frac{N}{2H}$	$P(\overline{A}) = S_{\overline{A}} = h(x_l - x_{min}) + h(x_{max} - x_u) =$ $= h(x_l - x_{min} + x_{max} - x_u) =$ $= h(x_{max} - x_{min} - [x_u - x_l]) = \frac{2H - N}{2H} = 1 - \frac{N}{2H}$

In this case, to calculate specific numerical values of probabilities (Table 1), it is necessary to know the corresponding distribution density $f_1(x)$ of the parameter x. If the distribution law of characteristic parameter x values is uniform and, accordingly, there are limits of possible values of this parameter (1-2), the probabilities of occurrence of control events (6) will depend on the ranges of parameter x values, which correspond to these events, and the value of the probability density $f_1(x)$ of distribution (Fig. 3).

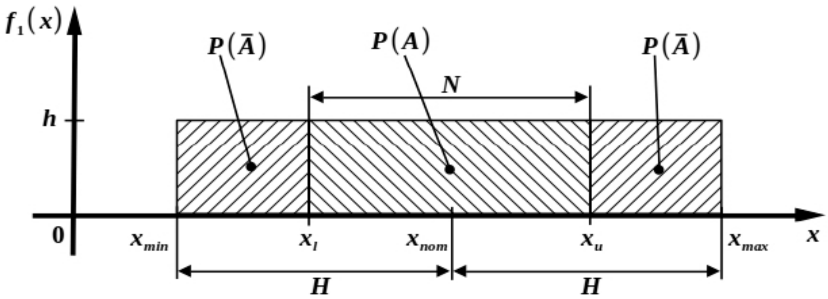

Fig. 3. Probability density $f_1(x)$ of the parameter x distribution

The probability density $f_1(x)$ of distribution of the characteristic parameter x of the technological process on the basis of (Fig. 3) will be constant $f_1(x) = h$. The expression for calculation of the distribution density $f_1(x)$ on the basis of (Fig. 3) and boundary values (1-2) will have the form:

$$f_1(x) = h = \frac{1}{x_{max} - x_{min}} = \frac{1}{x_{nom} + H - (x_{nom} - H)} = \frac{1}{2H}. \quad (8)$$

The values of probabilities (7) will be calculated on the basis of integrals of probability densities for integration ranges (1-2) and will have the form (Table 1). Using the probability density function of the parameter x (8) we can obtain the corresponding expressions of probabilities (7), the calculation of which is presented in Table 1. As can be seen, the expressions obtained correspond to condition (7).

On the other hand, the integral calculates the area under the probability density $f_1(x)$ line and between the boundary values (1-2). Under the uniform law, this probability density $f_1(x)$ is described by a straight line that is parallel to the parameter x value axis (Fig. 3). As a result, the procedure for calculating the integral can be replaced by calculating the areas of rectangles (Fig. 3) $P(A) = S_A$ and $P(\overline{A}) = S_{\overline{A}}$. The corresponding calculations are summarized in Table 1.

3.2. Presence of measurement uncertainty

The presence of uncertainty in the measurement of characteristic parameters of the technological process leads to the appearance of uncertainty in the assessment of compliance of the technological process with the norms.

Measurement errors of the characteristic parameter x of the technological process by real measuring devices lead to the appearance of deviation of the measurement result z of the parameter x from its actual value. Therefore, the measurement result z will depend not only on the parameter itself x (3), but also on the measurement error y of this parameter. In this paper, we will consider only the random additive error of measurement (4).

Let us assume in the paper that the random measurement error y has a uniform distribution law (Fig. 4) with probability density $f_2(y)$:

$$f_2(y) = \frac{1}{2\mu}, \quad (9)$$

where μ is the maximum value of the measurement error

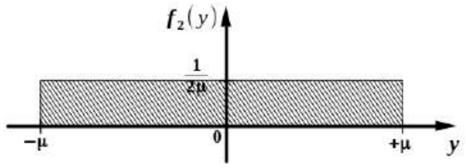

Fig. 4. Density distribution function of measurement error

The presence of measurement error y leads to the fact that as a result of control the decision made about the compliance of the technological process with the norms may diverge from the actual state of the process. Thus, the result of the process control procedure leads to the following elementary events:

- event B: the technological process is normal and complies with the standards:
- event $\overline{B}$: the technological process is faulty and does not comply with the standards.

Since the measured value z should correspond to the given norms (1), the calculations of the probability of occurrence of events B and $\overline{B}$ coincide with the expressions from Table 1. The integration ranges will accordingly be the same as for x (Fig. 3).

Since the actual state of the technological process (Tab. 1) may differ from the control result, the result is combinations of elementary events, the set of variants of which can be described by the following expression:

$$\Omega = \{AB, \overline{AB}, A\overline{B}, \overline{A}B\}. \quad (10)$$

The significance of each event (10) can be characterized by:

- the AB control result corresponds to the actual state that the process conforms to the norms,
- the $\overline{AB}$ control result corresponds to the actual state that the process does not correspond to the norms,
- the $A\overline{B}$ result of the control does not correspond to the actual state that the process is normal, but the result of the control shows that the process does not correspond to the norms,
- the $\overline{A}B$ result of control does not correspond to the actual state that the process does not correspond to norms, but in the result of control, the decision corresponds to norms.

The outcomes (10) constitute a complete group of mutually exclusive events (Table 2):

$$\sum_{i=1}^{4} P_i = 1. \quad (11)$$

The set of outcomes that lead to the occurrence of events (10) are obtained by finding the values of the characteristic parameter x of the technological process and the measurement result z of this parameter during the control of the technological process in the corresponding ranges (1-2). Probabilities (11) of occurrence of events (10) as a result of control will be proportional to the intersections of ranges (1-2) that correspond to the parameter x and the measurement result z (Table 2).

The probability density of the probability distribution of the values of the control results z will be a function of the densities of the characteristic parameter x and the measurement error y: $f_3(z) = f_3(f_1(x), f_2(y))$. In addition, it is necessary to consider the joint densities of distributions $f_4(x,z)$ and $f_5(x,y)$. The area of integration for finding probabilities (Table 2) will be the plane in coordinates x,z (Fig. 5). Intersections of the ranges (1-2) give corresponding rectangular plane geometric figures (Fig.5), the areas of which are proportional to the probability of occurrence of the corresponding event (10). The areas of geometric figures that contribute to the occurrence of complex events are estimated from Fig. 5.

The probability density $f_1(x)$ of the parameter x value distribution and the density $f_3(z)$ of its measurement result z are dependent, as well as the corresponding events A and B, because of the dependence of the measurement result on the parameter x (3). In order to pass independent events, it is necessary to substitute in the expressions in Table 2 instead of z its definition from expression (4). After the mathematical transformation, the probabilities from Table 2 will be represented by the expressions given in Table 3.

Table 2. Probabilities of occurrence of complex events (10)

	A	$\overline{A}$
B	$P(AB) =$ $= P_1[(x_l \le x \le x_u) \cap (x_l \le z \le x_u)]$	$P(\overline{A}B) =$ $= P_4\begin{bmatrix}([x_{min} \le x < x_l] \cup [x_u < x \le x_{max}]) \cap \\ \cap\ (x_l \le z \le x_u)\end{bmatrix}$
$\overline{B}$	$P(A\overline{B}) =$ $= P_3\begin{bmatrix}(x_l \le x \le x_u) \cap \\ \cap\ ([x_{min} \le z < x_l] \cup [x_u < z \le x_{max}])\end{bmatrix}$	$P(\overline{AB}) =$ $= P_2\begin{bmatrix}([x_{min} \le x < x_l] \cup [x_u < x \le x_{max}]) \cap \\ \cap\ ([x_{min} \le z < x_l] \cup [x_u < z \le x_{max}])\end{bmatrix}$

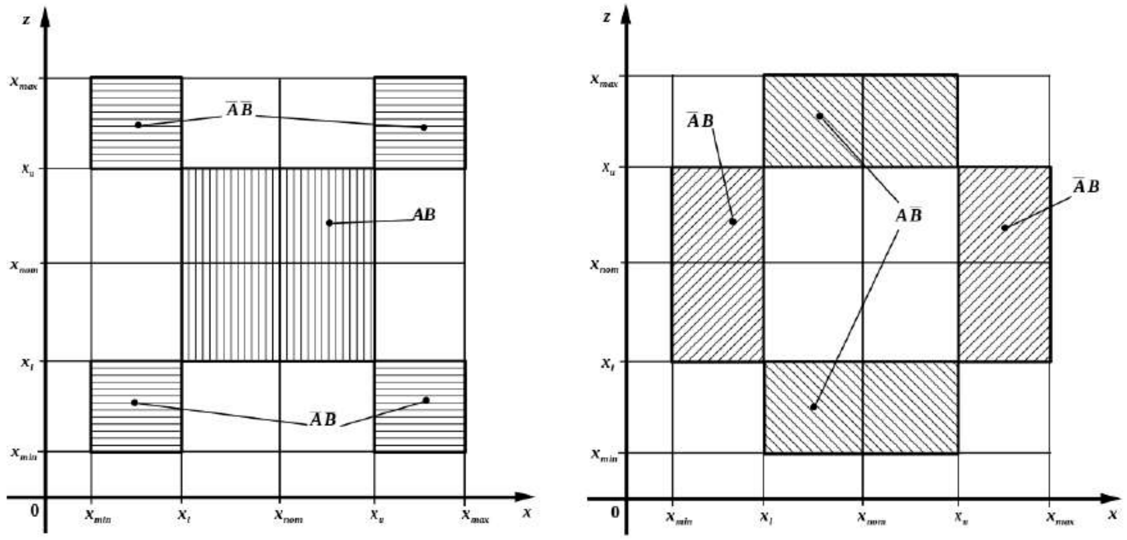

Fig. 5. Integration areas for finding probabilities (Table 2)

Table 3. Probabilities of the occurrence of independent events (10)

	A	$\overline{A}$
B	$P(AB) = P_1[(x_l \le x \le x_u) \cap (x_l \le z \le x_u)]=$ $=P_1[(x_l \le x \le x_u) \cap (x_l \le x + y \le x_u)]=$ $=P_1[(x_l \le x \le x_u) \cap (x_l - x \le y \le x_u - x)]$	$P(\overline{A}B) = P_4\begin{bmatrix}([x_{min} \le x < x_l] \cup [x_u < x \le x_{max}]) \cap \\ \cap\ (x_l \le z \le x_u)\end{bmatrix}=$ $=P_4\begin{bmatrix}([x_{min} \le x < x_l] \cup [x_u < x \le x_{max}]) \cap \\ \cap\ (x_l \le x + y \le x_u)\end{bmatrix}=$ $=P_4\begin{bmatrix}([x_{min} \le x < x_l] \cup [x_u < x \le x_{max}]) \cap \\ \cap\ (x_l - x \le y \le x_u - x)\end{bmatrix}$
$\overline{B}$	$P(A\overline{B}) = P_3\begin{bmatrix}(x_l \le x \le x_u) \cap \\ \cap\ ([x_{min} \le z < x_l] \cup [x_u < z \le x_{max}])\end{bmatrix} =$ $P_3\begin{bmatrix}(x_l \le x \le x_u) \cap \\ \cap\ ([x_{min} \le x + y < x_l] \cup [x_u < x + y \le x_{max}])\end{bmatrix} =$ $P_3\begin{bmatrix}(x_l \le x \le x_u) \cap \\ \cap\ ([x_{min} - x \le y < x_l - x] \cup [x_u - x < y \le x_{max} - x])\end{bmatrix}$	$P(\overline{AB}) = P_2\begin{bmatrix}([x_{min} \le x < x_l] \cup [x_u < x \le x_{max}]) \cap \\ \cap\ ([x_{min} \le z < x_l] \cup [x_u < z \le x_{max}])\end{bmatrix}=$ $=P_2\begin{bmatrix}([x_{min} \le x < x_l] \cup [x_u < x \le x_{max}]) \cap \\ \cap\ ([x_{min} \le x + y < x_l] \cup [x_u < x + y \le x_{max}])\end{bmatrix}=$ $=P_2\begin{bmatrix}([x_{min} \le x < x_l] \cup [x_u < x \le x_{max}]) \cap \\ \cap\ ([x_{min} - x \le y < x_l - x] \cup [x_u - x < y \le x_{max} - x])\end{bmatrix}$

In the case of independent variables x and y for the calculation of probabilities (Table 3), the areas of integration for calculating the probabilities of complex events (10) become more complicated. In addition to the boundary values of the intervals introduced above, there appear limiting functions passing through the boundary points (1-2)

$$y = x_{min} - x, \quad y = x_{max} - x, \quad y = x_l - x, \quad \text{and} \quad y = x_u - x.$$

In the case of uniform distribution laws of independent random variables x and y, to calculate probabilities (Table 3) we have the joint distribution density $f_5(x, y)$, which will be equal to the product of distribution densities of the characteristic parameter $f_1(x)$ and the additive measurement error $f_2(y)$:

$$f_5(x, y) = f_1(x) f_2(y). \tag{12}$$

The difficulty in calculating the probabilities of occurrence of the outcomes of the control result (Table 3) using the procedures of integration of the joint distribution density (12) was related to the construction of the ranges of integration of the probability densities of independent random variables x and y, as well as integration of expression (12). Despite the fact that the areas of integration (Fig. 6) of probabilities of occurrence of events (Table 3) represent flat figures bounded by straight lines, the determination of specific ranges of integration presents some difficulty and requires a certain number of computational operations to calculate integrals.

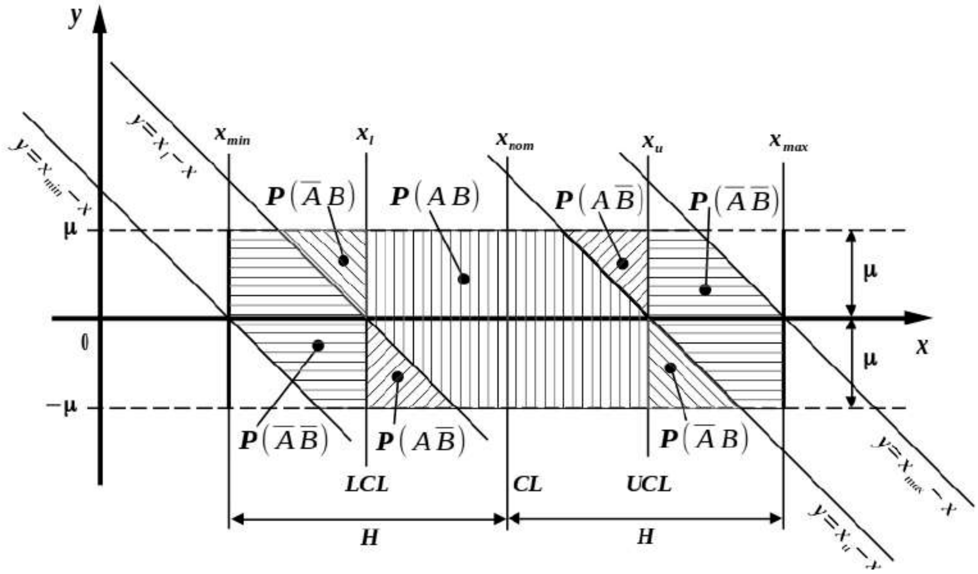

Fig. 6. Integration areas for independent quantities x and y from Table 3

Integration of the corresponding areas, which contribute to the occurrence of events (10), for the entire range of values x and y, involves certain difficulties, which are associated with the shapes of geometric figures. In addition, the measurement errors will have the greatest influence on the assessment of compliance of the technological process with the norms at the boundaries of the range of acceptable values (1). Therefore, it is expedient to evaluate the compliance of the technological process with the norms at the boundaries of the range of permissible values (1).

False alarm $A\overline{B}$ occurs because of the true value of the parameter x, which is in the zone of permissible values (1), possible values of the random variable y (measurement error) are added during measurement and, as a consequence, the value of the measurement result z may be outside the area of permissible values. As a result, the event $\overline{B}$ of the control result of the technological process that it does not meet the norms occurs. Moreover, the greatest impact of measurement error will be when the characteristic parameter takes the value $x = x_u$. Thus, there is a certain value η, at which the impact of measurement error y can be neglected. For a uniform distribution law of the measurement error y (9), this value can be taken as $\eta = \mu$. This value η can be used to introduce additional control limits on both sides of the boundary values (1). I.e. at values of characteristic parameter of technological process equal to the boundary values $x = x_l$ or $x = x_u$ the measurement error will have the greatest influence. The values of additional control limits for the boundary values (1) are given in Table 4.

Table 4. Additional control limits

$x = x_l$	$x = x_u$
$x_l' = x_l - \eta,$ $x_l'' = x_l + \eta.$	$x_u' = x_u - \eta,$ $x_u'' = x_u + \eta.$

Introduction of additional control boundaries allows to reduce the volume of control and measuring operations without loss of statistical reliability of the decisions made. Graphical location of additional control boundaries (Table 4) in relation to the boundary values of the interval of acceptable values (1) is shown in Fig. 7.

The control limits are located symmetrically with respect to the lower and upper LCL and UCL action lines of the control chart, which corresponds to the boundary value (1) of the range of permissible values (Fig. 7).

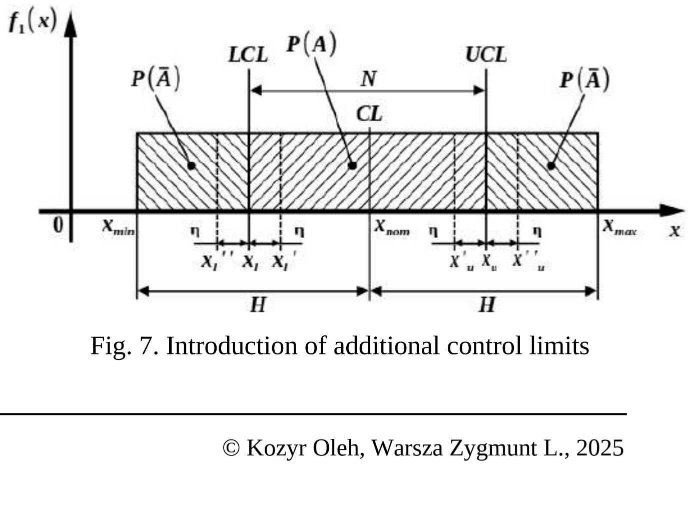

Fig. 7. Introduction of additional control limits

4. Assessing the plausibility of the result of process control

In the article two estimations of the result of the technological process control were used: a priori and posteriori. A priori estimation of the control result characterizes the reliability of the control result, i.e. the conformity of the adopted decision to the actual state of the process. This estimate is calculated before the control procedure based on the known distribution laws of the characteristic parameter x of the technological process and measurement error y. In addition, the main source of a priori information are standards and norms that are used in industry for a given technological process.

A posteriori estimation of the control result characterizes the plausibility of this result, i.e. confidence in the accepted result or the probability of making a correct decision. This estimation is performed after the control procedure, when the control result is known. In this case, two events occur as a result of control: the process complies with norms B and does not comply with the norms $\overline{B}$. Each of these events is favored by two events from the set of outcomes (10). Thus, the event B is favored by events (10):

$$P(B) = P(AB) + P(\overline{A}B). \qquad (13)$$

The event $\overline{B}$ is favored by the following events (10):

$$P(\overline{B}) = P(A\overline{B}) + P(\overline{AB}). \qquad (14)$$

Thus, any control outcome (13-14) will be favored by only two events (10). A posteriori estimation of the control outcome (13-14) allows us to reduce the set of possible events (10) by half and consider only those events that favor it.

We will further consider the probability of making a false decision on the compliance of the technological process with norms. In other words, we will estimate the probability of erroneous decisions:

- $P(A\overline{B})$ probability of the false alarm,
- $P(\overline{A}B)$ probability of the undetected alarm.

The a-priori probability of occurrence of any of the control outcomes (10) corresponds to the product of the unconditional probability of one of the events by the conditional probability of the other, provided that the first event occurred (Table 5) [4].

Let us consider the reliability of estimation of the control result of non-compliance with the norms $\overline{B}$. It takes place under the following circumstances [14]:

• The object actually complies with the norms A (the technological process is in a statically controlled state), and the evaluation result, after the measurement procedure, carries false information. As a result, a decision about non-compliance with the norms $\overline{B}$ is made. That is, the influence of measurement error y leads to the occurrence of false alarm the probability of which is determined (Table 5);

• The object in reality does not correspond to the norms $\overline{A}$ and as a result of evaluation the decision about non-compliance $\overline{B}$ is made, which correctly reflects its functional state. That is, although the error y affects the measurement result z, but there is no erroneous decision (Table 5).

Table 5. A priori probabilities of conditional events (13-14)

	A	$\overline{A}$
B	$P(AB) = P(A)P(B/A)$	$P(\overline{A}B) = P(\overline{A})P(B/\overline{A})$
$\overline{B}$	$P(A\overline{B}) = P(A)P(\overline{B}/A)$	$P(\overline{AB}) = P(\overline{A})P(\overline{B}/\overline{A})$

In Table 5, in the expressions for determining the probabilities of complex events (10), the conditional probabilities of the following events are given:

• $P(B/A)$ the correct conditional probability of deciding on the conformity of the technological process to the norms;

• $P(\overline{B}/\overline{A})$ correct conditional probability of deciding about non-compliance of the technological process with the norms;

• $P(\overline{B}/A)$ false conditional probability of deciding that the technological process does not comply with the norms, while it does comply with the norms;

• $P(B/\overline{A})$ false conditional probability of deciding that a technological process conforms to norms while it does not conform to norms.

Multiplication of the absolute probability of an actual compliance event by the conditional probability of the process control outcome (Table 5) is the a-priori probabilities of inconsistent events (10). For example, $P(A)P(\overline{B}/A)$ is the a priori probability of a false alarm.

Let us perform a posteriori estimation of the false decision probability for both events B and $\overline{B}$ using the Bayesian approach. Let us evaluate the plausibility of the obtained result. That is, consider what part of the false outcome is present in the decision. Based on the probability of the occurrence of the outcome B (13) and conditional events (Table 5), we estimate what fraction of the false decision (undetected alarm) is present in the result B (Table 6). Also, the fraction of false alarm in the decision $\overline{B}$ (14) when considering the conditional events (Table 5), i.e. the plausibility of this result, is given in (Table 6).

An undetected alarm, that is, the technological process does not actually meet the norm $\overline{A}$, and the solution, after the conformity assessment procedure, is recognized as suitable B. But the evaluation result B may appear when the object actually complies with the norm A. Thus, in order to assess the plausibility of the obtained result B of the evaluation of non-compliance with norms, it is necessary to:

• estimate the a priori probability of an undetected failure $P(A\overline{B})$;

• estimate how much we can trust the obtained result about suitability, i.e. to determine what part of the erroneous decision as a result B.

Table 6. A posterior probabilities of false decisions

B	$\overline{B}$
$P(\overline{A}/B)=\frac{P(\overline{A}B)}{P(B)}=\frac{P(\overline{A}B)}{P(\overline{A}B)+P(AB)}=$ $\frac{P(\overline{A})P(B/\overline{A})}{P(\overline{A})P(B/\overline{A})+P(A)P(B/A)}=\frac{1}{1+\frac{P(A)P(B/A)}{P(\overline{A})P(B/\overline{A})}}=\frac{1}{1+k_B},$	$P(A/\overline{B})=\frac{P(A\overline{B})}{P(\overline{B})}=\frac{P(A\overline{B})}{P(A\overline{B})+P(\overline{AB})}=$ $=\frac{P(A)P(\overline{B}/A)}{P(A)P(\overline{B}/A)+P(\overline{A})P(\overline{B}/\overline{A})}=\frac{1}{1+\frac{P(\overline{A})P(\overline{B}/\overline{A})}{P(A)P(\overline{B}/A)}}=\frac{1}{1+k_{\overline{B}}},$
$k_B=\frac{P(A)P(B/A)}{P(\overline{A})P(B/\overline{A})}.$	$k_{\overline{B}}=\frac{P(\overline{A})P(\overline{B}/\overline{A})}{P(A)P(\overline{B}/A)}.$

The posterior probability $P(\overline{A}/B)$ represents confidence in the result B obtained that the process complies with norms or the posterior probability of the plausibility of the compliance decision (Table 6).

The same applies to the posterior probability of nonconformity $P(A/\overline{B})$, which evaluates the plausibility of the decision on nonconformity (Table 6). In Table 6, for the control outcome $\overline{B}$ in the numerator is the absolute a priori probability of receiving a false decision on nonconformity. In the denominator there is the sum of probability of complex events forming the outcome $\overline{B}$. Thus, the probability $P(A/\overline{B})$ shows what share of the erroneous decision on non-compliance of the technological process with the norms makes up in the accepted decision $\overline{B}$, i.e. corresponds to the confidence in the obtained result of conformity assessment.

The posterior probability as well as any probability varies in range 0.1. Let's consider the limit values of expressions for posterior probabilities (Table 5). Let's consider for example the limit values of the posterior probability of false alarm $P(A/\overline{B})$ (Table 6):

- $P(A/\overline{B})=0$ a-posteriori probability that there is no false alarm, i.e. the technological process really does not comply with the norms. This is possible when the coefficient $k_{\overline{B}}$ tends to infinity $lim_{k_{\overline{B}}\to\infty}\frac{1}{1+k_{\overline{B}}}=0$. The coefficient $k_{\overline{B}}=\frac{P(\overline{AB})}{P(A\overline{B})}\to\infty$ tends to infinity, $lim_{P(A\overline{B})\to 0}\frac{P(\overline{AB})}{P(A\overline{B})}=\infty$when the a priori joint probability of false alarms $P(A\overline{B})$ is zero;
- $P(A/\overline{B})=1$ the a posteriori probability of the presence of a false alarm, i.e. the technological process really complies with the norms. In this case, the coefficient $k_{\overline{B}}$ will be zero $\frac{1}{1+k_{\overline{B}}}=1\Rightarrow 1+k_{\overline{B}}=1\Rightarrow$ $k_{\overline{B}}=0$. The zero value of the coefficient $k_{\overline{B}}=\frac{P(\overline{AB})}{P(A\overline{B})}=$ 0 is possible in case the a priori joint probability of non-compliance of the technological process is equal to zero $P(\overline{AB})=0$.

The same can be said for the posterior probability of undetected alarm $P(\overline{A}/B)$.

5. Plausibility estimation of the control result in case of uniform distribution law

Reliability of process control is an assessment of the possibility of occurrence of one of the four events (10) of the process control result. This assessment is an a priori estimate of the correctness of decision making about the conformity of the technological process to the norms (Table 3). This assessment allows to analyze the probability of occurrence of false outcomes in the result of control based on a priori information about the distribution densities of the characteristic parameter of the technological process and random additive error of measurement. Appropriate standards are also used to determine integration ranges for calculating the corresponding probabilities of complex events (Table 3).

The paper considers the procedure of estimation of the influence of measurement error on the occurrence of an erroneous decision as a result of control, namely false alarm $A\overline{B}$. In addition, due to the reasons mentioned above, we will consider conformity assessment for the upper boundary of the range of permissible values x_u.

5.1. A priori probability of false alarms

A priori probabilities of false outcomes of the control result (Table 3) are calculated on the basis of integration of the ranges of values that favor the occurrence of these false events (Fig. 6) using the boundary values of the ranges of values of the characteristic parameter and the measurement result (1-2). Since the measurement error has the greatest influence on the boundaries of the range of acceptable values of the characteristic parameter of the technological process, we will calculate the probabilities of false events (Table 5) at the boundaries of the acceptable range (1). Let's calculate the a priori probability of false alarm $P(A\overline{B})$ for the upper boundary of the permissible range x_u. In the calculations we will consider the introduced additional control boundaries (Table 4) to determine the integration ranges of the probability density function of the characteristic parameter distribution (Fig. 8).

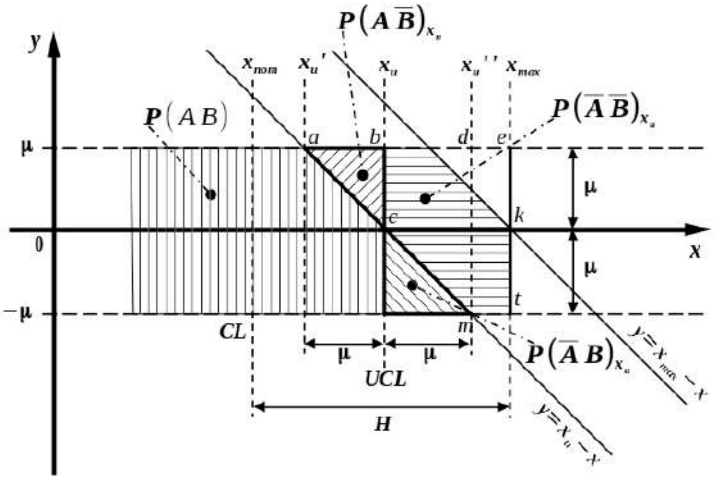

Figure 8. Integration ranges of probability density (12)

It follows from Fig. 8 and Table 4 that under the condition $x < x_u'$, the probability of false failure can be neglected. Only, starting from $x > x_u'$, it is possible that the measurement result will be $z > x_u$, i.e. a decision of non-conformity can be made. For this case, the expression of the probability of making a false decision about the non-compliance of the technological process with the norms (Table 3) will have the form:

$$P(A\overline{B})_{x_u} = P_3[(x_u' \le x \le x_u) \cap (x_u - x < y \le +\mu)] =$$
$$= P_3[(x_u - \mu \le x \le x_u) \cap (x_u - x < y \le +\mu)]. \quad (15)$$

Let us reflect in the orthogonal coordinate system the region where the intersection of the x and y regions can lead to a false alarm (15). In Fig. 8, this region is shaded with a right-hand slope.The area of the isosceles triangle *a b c* in Fig. 8 corresponds to the false rejection. This area allows us to determine the limits of integration of the joint distribution density of the characteristic parameter x of the technological process and measurement error y (12). According to expression (15), the a priori probability of false alarm will be:

$$P(A\overline{B})_{x_u} = \int_{x_u-\mu}^{x_u} f_1(x)\left(\int_{x_u-x}^{+\mu} f_2(y)dy\right)dx. \quad (16)$$

Using the accepted distribution densities of the parameter x and measurement error y (8-9) we calculate the a priori probability (16):

$$P(A\overline{B})_{x_u} = \int_{x_u-\mu}^{x_u} f_1(x)\left(\int_{x_u-x}^{+\mu} f_2(y)dy\right)dx=$$
$$= \int_{x_u-\mu}^{x_u} \frac{1}{2H}\left(\int_{x_u-x}^{+\mu} \frac{1}{2\mu}dy\right)dx =$$
$$= \frac{1}{4H\mu}\int_{x_u-\mu}^{x_u}\left(\int_{x_u-x}^{+\mu} dy\right)dx. \quad (17)$$

The result of the calculations gave the expression (17), which consists of the product of the constant $\frac{1}{4H\mu}$ over the integrals $\int_{x_u-\mu}^{x_u}\left(\int_{x_u-x}^{+\mu} dy\right)dx$. Let us calculate these integrals starting from the inner integral on y:

$$\int_{x_u-x}^{+\mu} dy = y|_{x_u-x}^{+\mu} = \mu - x_u + x. \quad (18)$$

As a result of the solution of the inner integral on y we obtain the equation of the straight line (18). This equation is integrated by the external integral on x:

$$\int_{x_u-\mu}^{x_u}(\mu - x_u + x)\,dx = (\mu - x_u)\int_{x_u-\mu}^{x_u} dx + \int_{x_u-\mu}^{x_u} x\,dx=$$
$$=(\mu - x_u)x|_{x_u-\mu}^{x_u} + \frac{X^2}{2}\Big|_{x_u-\mu}^{x_u} = (\mu - x_u)\mu + \frac{x_u^2-(x_u-\mu)^2}{2}=$$
$$=\frac{2\mu^2-2x_u\mu+x_u^2-x_u^2+2x_u\mu-\mu^2}{2} = \frac{\mu^2}{2}. \quad (19)$$

As a result of calculations of integrals on x and on y (18-19) we obtained

$$\int_{x_u-\mu}^{x_u}\left(\int_{x_u-x}^{+\mu} dy\right)dx = \frac{\mu^2}{2}. \quad (20)$$

Thus, the a priori false alarm probability (16) as a result of integrating the joint probability density function over the ranges (Fig. 8) based on the obtained expressions (17-20) will be as follows:

$$P(A\overline{B})_{x_u} = \frac{1}{4H\mu}\int_{x_u-\mu}^{x_u}\left(\int_{x_u-x}^{+\mu} dy\right)dx = \frac{1}{4H\mu}\frac{\mu^2}{2} = \frac{\mu}{8H}. \quad (21)$$

The calculation of the a priori probability of false alarms or confidence of control using the joint probability density integration procedure (12) was a relatively simple procedure (21). The calculation of other a-priori probabilities (Table 3) involves significant difficulties in setting the ranges of integration of the joint probability density function (12) and then calculating the corresponding integrals. A simplified calculation procedure is proposed in the paper.

The integration of the joint probability density function (12) to find the a priori probabilities (Table 3) will in all cases have a form similar to the obtained expression (17). It consists of the product of the constant $\frac{1}{4H\mu}$ over the integrals (20). The a-priori probabilities from Table 3 will differ only in the expressions of the integration ranges of the joint probability density function (12). Therefore, we take expression (17) as the basis for all expressions for calculating the a-priori probabilities (Table 3). Let us introduce notations for the constant in expression (17):

$$L = \frac{1}{4H\mu}. \quad (22)$$

When calculating the probabilities (Table 3), this constant (22) will be always present. It remains to calculate the integrals (20) for the remaining probabilities from Table 3. Since the integration areas of the corresponding probabilities (Table 3) are represented by plane shapes (Fig. 8), and integrals are by definition the areas under some curve, let us find the areas of these integration areas. Let us calculate the area of the isosceles triangle *abc* (Fig. 8) for the a priori probability of a false alarm, since we have already calculated this probability using integration. The area of triangle *abc* will be:

$$S_{abc} = \frac{(x_u - x_u{}')\mu}{2} = \frac{[x_u - (x_u - \mu)]\mu}{2} = \frac{\mu^2}{2}. \quad (23)$$

If we compare the expressions for the area of the integration region of the false alarm probability (23) with the results of calculating the integrals (20), we can see that they are equal to each other. Thus, when calculating a-priori probabilities (Table 3), we can replace the calculation of integrals (20), with the corresponding integration ranges, by the calculation of the areas of the corresponding integration areas (Fig. 8).

As a result, the calculation of a-priori probabilities (Table 3) can be represented by the product of the constant L, which is the product of the values of the probability densities of the parameter x and the measurement error y, by the areas of the corresponding integration region. In the case of a priori probability of false alarm, this expression will have the form:

$$P\left(A\overline{B}\right)_{x_u} = LS_{abc} = \frac{1}{4H\mu}\frac{\mu^2}{2} = \frac{\mu}{8H}. \quad (24)$$

Geometric interpretation of probabilities of complex events (Table 3) are volumes of prisms (24), the heights L of which are the same and are the product of probability densities of random variables x and y, and the bases are the areas of integration regions of these probabilities Fig. 8.

The a priori probability of making a correct decision about non-compliance of the technological process with the norms (Table 3) on the basis of expression (24) will look as follows:

$$P\left(\overline{AB}\right)_{x_u} = LS_{bdektmc} = \frac{1}{4H\mu}\frac{4\mu H - 4\mu x_u - \mu^2}{2}. \quad (25)$$

5.2. A posteriori probability of false alarms

Let us obtain expressions for estimating the likelihood of the outcome of a false alarm of process control. This estimate is quantitatively represented by the posterior probability of a false alarm. According to the Bayesian approach, the credibility of the decision that the process is out of compliance, given that the true state of the process complies, is presented in Table 6. In Table 6, the expressions for calculating the posterior probability are presented for calculating the total false alarm probability. And since we consider the probability of false alarm for the case of influence of measurement error y on the measurement result z of the parameter x at the upper boundary of the range of acceptable values x_u, the expression of the posterior probability of false alarm (Table 6) in this case will be:

$$P\left(A/\overline{B}\right)_{x_u} = \frac{P\left(A\overline{B}\right)_{x_u}}{P\left(A\overline{B}\right)_{x_u} + P\left(\overline{AB}\right)_{x_u}}. \quad (26)$$

In our case, when the laws are uniform (8-9), the complex event probabilities (Table 3) correspond to the prism volume (24-25) with height (22) and base areas (23) according to Fig. 8. Accordingly, the value of the a-posteriori probability of non-compliance of the process with the norms can be written by the expression:

$$P\left(A/\overline{B}\right)_{x_u} = \frac{LS_{abc}}{LS_{abc} + LS_{bdektmc}} = \frac{S_{abc}}{S_{abc} + S_{bdektmc}}. \quad (27)$$

To establish the plausibility of the erroneous decision about the non-compliance of the technological process with the norms (26), we will calculate the areas of the bases of the corresponding prisms (27). We will consider the area corresponding to the a priori probability of erroneous alarm (23) and the area that corresponds to the a-priori probability of making a correct decision about non-compliance of the technological process with the norms $S_{bdektmc}$. Based on the expression (27) we obtain:

$$P\left(A/\overline{B}\right)_{x_u} = \frac{S_{abc}}{S_{abc} + S_{bdektmc}} = \\ = \frac{\frac{\mu^2}{2}}{\frac{\mu^2}{2} + \frac{4\mu H - 4\mu x_u - \mu^2}{2}} = \frac{\mu}{4(H - x_u)}. \quad (28)$$

By calculating the posterior probability (plausibility) of making an erroneous decision (28), we increase the probability of assessing compliance of the technological process with the norms by performing a statistical evaluation of the erroneous failure, i.e., we clarify the erroneous failure. Like any probability, expression (28) must fulfill the condition:

$$0 \le \frac{\mu}{4(H - x_u)} \le 1. \quad (29)$$

Let us consider the extreme cases of condition (28). When the a posteriori probability of making an erroneous decision about the non-compliance of the technological process with the norms is:

- $P\left(A/\overline{B}\right)_{x_u} = 0$ it is possible in the case when $\mu = 0$, that is, the absence of additive random measurement error, and hence the absence of false rejection (21);
- $P\left(A/\overline{B}\right)_{x_u} = 1$ in this case $\frac{P\left(A\overline{B}\right)_{x_u}}{P\left(A\overline{B}\right)_{x_u} + P\left(\overline{AB}\right)_{x_u}} = 1 \Rightarrow P\left(A\overline{B}\right)_{x_u} \equiv P\left(A\overline{B}\right)_{x_u} + P\left(\overline{AB}\right)_{x_u}$ and it is possible if there is no probability $P\left(\overline{AB}\right)_{\mu}$ and means that in expression (26) in the denominator the probability of

correct decision about non-compliance of technological process with the norms $P(\overline{AB})_\mu = 0$, that is, it is a valid case of false failure. Otherwise, we can say that the probability of the non-compliance process asymptotically approaches zero.

Let us consider a numerical example of assessing the plausibility of the decision made about the non-compliance of the technological process with the norms, i.e. the event $\overline{B}$. As already mentioned, both false alarms and actual non-compliance of the technological process with the norms contribute to the occurrence of the event $\overline{B}$ (14). For calculations we will use the obtained formulas for calculating the a priori probability of false alarm (24) and it's a-posteriori probability (28). For the uniform law of distribution of the characteristic parameter of the technological process x and measurement error y, the corresponding boundary values (1,8-9) are specified. For convenience of analysis, let us consider not the absolute values of these boundaries, but those reduced to the value of the half range H of possible values of the characteristic parameter x. Let us introduce the following relations:

- $\frac{\mu}{H} = \delta\mu$ relative additive error of measurement reduced to the half range of possible values of the characteristic parameter,
- $\frac{x_u}{H} = \delta x_u$ upper limit of the parameter reduced to the half range of possible values of the characteristic parameter.

Then, considering the introduced relations, let us rewrite probability expressions (24) and (28) as:

$$P(A\overline{B})_\mu = \frac{\delta\mu}{8}, \tag{30}$$

and

$$P(A/\overline{B})_\mu = \frac{\delta\mu}{4(1-\delta x_u)}. \tag{31}$$

Let us introduce the length of the range of unacceptable values h_u of the parameter x at the upper boundary x_u reduced to the half range of possible values H: $\delta h_u = 1 - \delta x_u$. Then, expression (31) will have the form:

$$P(A/\overline{B})_\mu = \frac{\delta\mu}{4\delta h_u}, \tag{32}$$

Let us consider the extreme cases for the expressions of a priori (30) and posterior (32) probabilities, considering that probabilities take values between 0 and 1. First consider the extreme cases for eq. (30):

- $P(A\overline{B})_\mu = 0$ there is no probability of a false alarm. This can be the case when there is no measurement error $P(A\overline{B})_\mu = \frac{\delta\mu}{8} = 0 \Rightarrow \delta\mu = 0$;
- $P(A\overline{B})_\mu = 1$. The result obtained is a false alarm. This can be the case in the following case $P(A\overline{B})_\mu = \frac{\delta\mu}{8} = 1 \Rightarrow \delta\mu = 8 \Rightarrow \frac{\mu}{H} = 8 \Rightarrow \mu = 8H$.

Since the probability of false alarm is an a-priori estimate $P(A\overline{B})_\mu$, it is not known in advance what the outcome of the control will be. Therefore, when we consider a priori probabilities, we deal with four possible events (10).

Consider the marginal values of the posterior probability of a false alarm (32). Since this estimation is performed after the occurrence of a control event, in this case the process non-compliance event $\overline{B}$, two of the four complex events (10) are favorable $A\overline{B}$ or $\overline{AB}$. The marginal values of expression (32) will be:

- $P(A/\overline{B})_\mu = 0$. The obtained result $\overline{B}$ is caused by the fact that the technological process is really broken and the event $\overline{AB}$ is true. In this case $P(A/\overline{B})_\mu = \frac{\delta\mu}{4\delta h_u} = 0 \Rightarrow \delta\mu = 0$;
- $P(A/\overline{B})_\mu = 1$. The obtained result $\overline{B}$ is a false alarm. In this case $P(A/\overline{B})_\mu = \frac{\delta\mu}{4\delta h_u} = 1 \Rightarrow \delta\mu = 4\delta h_u \Rightarrow \frac{\mu}{H} = 4(1-\delta x_u) \Rightarrow \mu = 4H\left(1-\frac{x_u}{H}\right) \Rightarrow \mu = 4(H - x_u)$. In absolute value of the error of measurement in the length of the interval of unacceptable values h_u will be $\frac{\mu}{H} = 4\delta h_u \Rightarrow \mu = 4H\delta h_u$. We can conclude that in the case of a really false alarm, the value of the measurement error should be 4 times the value of the range of unacceptable values of the parameter x.

Let us consider a numerical example for given measurement error at the level of 10%, i.e. $\delta\mu = 0.1$. Then the a-priori probability of false alarm will be $P(A\overline{B})_\mu = \frac{\delta\mu}{8} = 0.0125$, i.e. 1.25%.

Let us consider how the a posteriori probability of false alarm will change in accordance with the increase in the value of the range of unacceptable values δh_u: $P(A/\overline{B})_\mu = f(\delta h_u)$. As we can see from (32) the value δh_u cannot be zero, because it would lead to an infinite value of the posterior probability, and the value of the probability cannot be greater than 1. This is logical, because if there is no range of unacceptable values δh_u, the posterior probability has no essence, because even in the presence of error, false alarm is still not possible. We have already defined an expression for the range of unacceptable values of the parameter x, so that the posterior probability of a false failure is 1, i.e. $\delta h_u = \frac{\delta\mu}{4}$. In the case of $\delta\mu = 0.1$ the range value is $\delta h_u = \frac{\delta\mu}{4} = \frac{0.1}{4} = 0.025$. These are the minimum values of the range of unacceptable values δh_u, so that the concept of posterior probability can be used. The maximum value of the range of unacceptable values h_u cannot be greater than H. In this case δh_u will be equal to $\delta h_u = 1$, that is, $h_u = H$. In other words, all values of the parameter will be invalid. This is not practical value to use. For this consideration, let's take the value δh_u equal to $\delta h_u = 0.6$. We obtain the values of posterior probabilities for the range of values $\delta h_u = 0.025 \ldots 0.6$. The dependence graph of $P(A/\overline{B})_\mu = f(\delta h_u)$ is shown in Fig. 9.

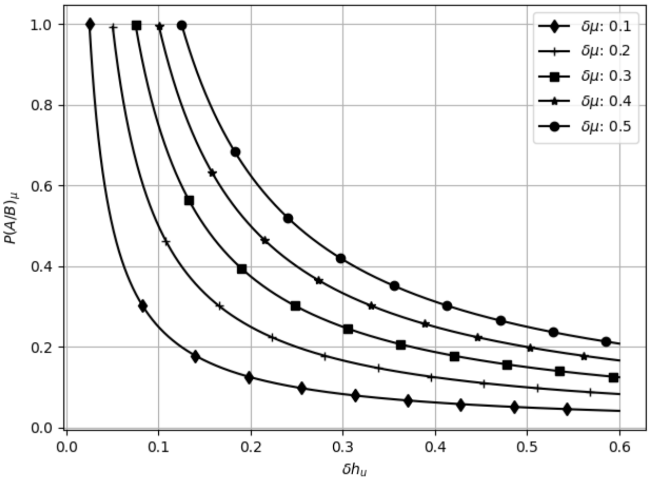

Figure 9. Posterior probabilities of false alarms

Let us study the influence of the reduced error $\delta\mu$ on the dependence $P\left(A/\overline{B}\right)_\mu = f(\delta h_u)$, i.e., let us consider the extended dependence $P\left(A/\overline{B}\right)_\mu = f(\delta h_u, \delta\mu)$ based on equation (32). Let us set the range of error values $\delta\mu = 0.1 \dots 0.5$. As a result, we obtain a family of curves (Fig. 9).

The following conclusions can be drawn from Fig. 9. As the range of unacceptable values δh_u increases, the probability of a false alarm $P\left(A/\overline{B}\right)_\mu$ decreases. This occurs due to the fact that the range of acceptable values H of the characteristic parameter decreases and the probability $P\left(\overline{AB}\right)_\mu$ increases accordingly. On the other hand, the greater the value of measurement error $\delta\mu$, the greater the probability $P\left(A/\overline{B}\right)_\mu$ and, accordingly, the lines of this probability shift upward and to the right, which can be seen from Fig. 9. Thus, the use of a posteriori probability or likelihood in estimating the state of the process during the control procedure allows us to refine the decision made. Namely, to clarify the probability of making a false decision about non-compliance of the object or process with the norms.

6. Summary and conclusions

This paper investigates the influence of imperfections of measuring instruments on the uncertainty of decision making about the conformity of technological process to norms. Calculations of two estimations of correctness of decision making on conformity are given: a priori and a posteriori or reliability and likelihood, respectively. The paper considers the uniform distribution of possible values of the characteristic parameter of the technological process and random additive error of its measurement.

The following results were obtained:

1. Calculations of two estimates of process compliance with norms using quantitative control charts were carried out. The expressions for estimation of reliability (a priori estimation) and plausibility (a posteriori estimation) of the accepted result of technological process control are given. The given expressions describe probability of obtaining a false alarm about non-compliance of technological process with the norms as a result of control, which provides measurement of characteristic features of processes before comparison with the norms. It is shown that measurement errors lead to erroneous decisions, namely to false alarms.

2. The paper proposes expressions for calculating a priori probabilities of control outcomes using independent events. Graphical representations of integration areas for calculating the corresponding probabilities of control outcomes are given. The paper proposes an approach to simplify the integration process by representing a priori probabilities of control outcomes by volumes of prisms with constant height and variable base. The process of integration of the joint probability density is proposed to be replaced by geometric calculation of the area of the shape that favors the occurrence of the corresponding control outcome.

3. Using the Bayesian approach, which is based on the solution obtained during the control, provides additional information and thus reduces by half the uncertainty due to possible combinations of elementary events.

4. The approach proposed by Prof. Volodarsky to assessing the plausibility of the result of process control can be used to control any technological process, where the state of the process is assessed on the basis of measuring the characteristic parameters of this process. This method allows considering the accepted control result to estimate its plausibility or the degree of confidence in this result.

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Статистична оцінка надійності рішень про стан керованого технологічного процесу на основі підходу Є. Володарського

Олег Козир, Зигмунт Л. Варшава

Анотація.

У статті розглядаються питання використання контрольних карт для дослідження параметрів, що описують стан продукції в процесі її виробництва. Обговорюється визначення надійності рішення на основі оцінки збурень, що в ній виникають. Розглянуто використання методу, запропонованого Євгеном Володарським, який базується на байєсівському підході. Розглянуто вплив похибок вимірювання та їх розподілу ймовірності на правильність прийнятих рішень. У статті розглядаються дві оцінки відповідності технологічного процесу нормам на основі результатів його контролю. Перша оцінка – це апріорна ймовірність або надійність результату контролю, яка виконується перед процедурою контролю та базується на апріорних даних про процес та похибці вимірювання. У статті пропонується використання другої оцінки, а саме апостеріорної ймовірності відповідності технологічного процесу нормам. Ця оцінка відповідності виконується після отримання результату контролю, коли для оцінки залишається лише половина набору елементарних подій, що сприяють виникненню одного з результатів контролю. Використання цієї оцінки дозволяє подвоїти статистичну достовірність оцінки результату контролю. Також визначається ефективність оцінки відповідності технологічного процесу встановленим нормам рівномірного розподілу значень його контрольованих параметрів та їх похибок вимірювання.

Ключові слова: статистичний контроль процесу, контроль якості процесу, похибки вимірювання, статистична достовірність рішень, оцінка відповідності, ймовірність Байєса, апріорна та апостеріорна ймовірність, моделювання даних на Python.