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A METHOD FOR NOISE ESTIMATION IN A MULTICHANNEL MEASUREMENT INFORMATION SYSTEMS USING SINGULAR VALUE DECOMPOSITION OF THE DATA MATRIX

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Abstract

Noise filtering is extensively applied in both the theoretical and practical aspects of signal processing. A much smaller number of scientific works is devoted to the extraction of noise from realizations of random processes in order to analyze them for specific tasks. The paper presents a method for separating signals and noise in multichannel measurement systems. The method utilizes the experimental data matrix and employs singular value decomposition (SVD) to analyze both the singular modes of the matrix and the partial matrices that comprise it. The conditions under which a partial first-order matrix describes signals in the system channels, and higher-order matrices contain noise components, have been determined. This requires the cosine of the angle between the data matrix and the first partial matrix must approach unity, and between the data matrix and the second matrix - to zero. Such conditions are achieved when the signal-to-noise ratio exceeds a threshold value. In some cases, the extracted noise can be utilized to determine measurement errors.

Keywords: data matrix, multichannel measurement system, noise filtering, singular value decomposition (SVD).

Introduction

In practical applications, multichannel measurement information systems (MMIS) are extensively used across various technical objects. These systems are capable of measuring either homogeneous parameters, such as pressure, or heterogeneous parameters, such as pressure, acceleration, and force. In the latter case, all physical quantities are normalized to ensure that parameters in all measurement channels are dimensionless. Experimental results, represented as realizations of random processes, contain data describing the physical process being measured, as well as noise. In most cases, the researcher is primarily interested only in the parameters of the random process. The presence of noise in measurement channels distorts the signals; however, in certain cases, its characteristics can provide useful information. In practice, it includes both internal and external noises, as well as random measurement errors, which are generally difficult to separate from the noise. The ideal situation would be one in which the main signals, noise, and measurement errors are fully separated. Measurement errors and noise characterize the uncertainty in the data. They are often closely related and may exhibit similar effects. In general, it is not possible to completely separate noise from measurement errors. The main reasons for this are the lack of sufficient information about the primary sources of errors and noise in the measurement system. Errors arise due to imperfections in measuring instruments, measurement methods, and the influence of external and internal factors. Random errors, in terms of their characteristics, are very similar to noise. Noise can originate from various sources (electrical, thermal, quantum, etc.) and exhibit different spectral characteristics. It can be either additive or multiplicative. The presence of nonlinear transformation functions in the measurement system can further distort the results. Thus, separating measurement errors from noise is a challenging task that requires a deep understanding of both the physical processes generating the data and the mathematical methods used for data processing. The choice of an appropriate method depends on the specific problem and requires experimental validation. In the following, random measurement errors and noise will be collectively referred to simply as “noise,” and we will analyze the possibility of separating the signal from the noise contained in multidimensional data.

The aim of this paper is to develop a method for assessing noise in a multichannel measurement information system based on singular value decomposition.

Problem Statement

Although complete separation of noise from measured results is impossible, there are a number of general methods that can reduce its impact. These methods include, in particular:

- Identification and analysis of possible sources of systematic and random errors;
- Calibration, i.e., comparing measurement results with reference values to determine and correct systematic errors;
- Performing repeated measurements and calculating the mean value to reduce the impact of noise on the signal;
- Using statistical data processing methods, including calculation of variance, standard deviation, confidence intervals to assess measurement accuracy, evaluation of skewness and multimodality, and checking distribution laws;
- Filtering to reduce the influence of noise components on the signal;
- Creating mathematical models of measurement processes to evaluate the impact of various factors on the results;

- Decomposition of data into empirical modes for analyzing non-stationary signals;
- Wavelet transforms for decomposing the signal into components with different frequencies and localizing signal features in time;
- Using a priori information about the signal and noise;
- Machine learning for pattern recognition in data and separating signal from noise;
- Clustering methods for dividing measurement data into groups with similar characteristics;
- Spectral analysis to identify periodic components that may be associated with systematic errors or external disturbances.

In many cases, it is advisable to combine different approaches and methods.

The general main drawbacks of existing noise filtering methods are:

- Loss of useful information, especially at low signal-to-noise ratios;
- Subjectivity in evaluating the quality of filtering due to the absence of a single universal criterion;
- Dependence of filtering efficiency on the type of noise present in the signal;
- Complexity in tuning filter parameters;
- Computational complexity, particularly for large data sets, such as in MMIS.

Each method has its limitations; for example, adaptive filters require a large amount of training data, while wavelet analysis is sensitive to the choice of decomposition basis. Recently, deep learning based on neural networks and adaptive methods grounded in data analysis, particularly Data Mining, have been employed. Analyzing large sets of data and filtering noise in MMIS requires consideration of alternative approaches, one of which, based on singular value decomposition, is presented in this paper.

Analysis of Recent Publications

A large number of scientific works are devoted to noise analysis in multichannel systems. In [1], a method for assessing the state of multichannel singular systems with multiplicative noise was developed based on singular value decomposition (SVD), taking into account dynamic and multiplicative noise, as well as measurement noise caused by measurement errors. The evaluation of multiplicative noise in the absence of information about input signals is carried out in [2] using a filter optimized according to the minimum mean square error criterion. The Kalman filter is also widely used in singular systems [3], where measurements consist of instantaneous and delayed observations, and the system includes multiplicative noise. In singular systems, the dynamics are described by a combination of algebraic and differential equations. The complex nature of singular systems poses many challenges for both analytical and numerical treatment of such systems [4]. Uncertainties in measurement systems are considered as multiplicative noise [5], and the least squares method is used for sensor optimization. Many articles have proposed algorithms for noise reduction. In [6], a filter based on SVD and the minimization of the mean square error (MSE) between the desired part of the signal and the sum of the filtered microphone signals is applied to improve speech intelligibility. Singular value decomposition and the principal component analysis (PCA) method have limited noise reduction capabilities under conditions of strong interference. For such conditions, multichannel SVD is used in [7] to obtain multiple signals constructed based on third-order tensors. Noise reduction is simplified when the system models are known, for which identification algorithms of multichannel measurements are developed in [8]. To reduce impulsive noise, a multichannel system for estimating damped sinusoids is proposed in [9]. Even a single-channel measurement system can be transformed into a virtual multichannel system, and SVD can then be applied [10] for “blind” signal separation. At the same time, alongside the use of SVD, it is advisable to reduce the dimensionality of the data matrix, as done, for example, in seismic signal processing in [11]. In [12], a method based on a modified Levinson algorithm is proposed, which does not require assumptions about the highest order of measurement channels with a finite impulse response. To improve the robustness of multichannel systems against modeling errors, a linear minimum variance estimator is described in [13]. The simplification of multivariate time series analysis is achieved through the use of the variational mode decomposition algorithm, which allows decomposing time series into several modes that possess specific spatial properties characteristic of a particular time series [14]. The SVD method is also applied to Hankel matrix sets for noise removal and normalization of the corresponding spaces [15]. To improve noise suppression efficiency in multichannel systems, the paper [16] proposes Regenerative Multidimensional Singular Value Decomposition, which maps measured signals into multidimensional data. The data is processed using Independent Component Analysis. Universal methods for analyzing signals with noise are becoming increasingly relevant. In [17], approaches for extending the use of adaptive Fourier decomposition with a predefined basis in multichannel systems are presented. The approach proposed in [18] for “blind” identification of autoregressive models uses the current autoregressive information model, which is extracted from correlation matrices. It does not require the Toeplitz channel convolution matrix, which is traditionally used in classical methods. At present, different types of noise are suppressed separately. In [19], a noise removal scheme is presented that takes into account their variance and signal-to-noise ratio. The scheme employs a threshold wavelet value and adaptive filtering of multi-source noise based on singular values. To reduce the impact of noise on deconvolution and improve image resolution, a multichannel deconvolution method is used [20].

Noise reduction methods using SVD are widely applied in various fields. For example, to extract fault features in technical systems and obtain their quantitative assessments, [21] proposes an algorithm for tensor SVD of multidimensional time series. The extraction of mechanical fault features based on multichannel measurement information systems (MMIS) is carried out in [22] using an adaptive tensor estimation model. The detection of damage in vibrating structures with different degrees of freedom is implemented in [23] based on recursive singular spectral analysis combined with autoregressive modeling. To extract fault signals from noise, [24] forms a tensor in the phase space and analyzes the principal components with suppressed noise based on tensor SVD. In [25], a new multichannel method for classifying mechanical fault signals is proposed, based on an extended quaternion singular spectrum. Here, quaternions are used to link signals from four channels. To address the mode-mixing problem in multidimensional empirical mode decomposition, [26] employs quaternion singular spectral analysis. It efficiently extracts the characteristic frequency of the fault from multi-channel signals.

Matrix SVD methods are applied in audio systems. In [27], a “blind” dereverberation method based on generalized spectral subtraction is used for noise suppression to improve speech recognition. To reduce vocal noise, [28] presents a comparative analysis of Wiener filtering, spectral subtraction, least squares methods, and digital filters. SVD methods are particularly intensively implemented in the medical field. In [29], it is shown that tensor decomposition of multichannel electroencephalography data can be used to analyze epileptic spikes. In [30], a method for reducing impulsive noise in electrical impedance tomography is described, replacing linear filters with an SVD-based decomposition filter. In [31], a model of multichannel skin conductance recording is developed for autonomic nervous system diagnostics, along with a multichannel deconvolution approach for sparse noisy data to generate reliable conclusions.

Information on blood pressure and other physiological parameters is obtained using multichannel sensors. To suppress noise in such systems, [32] proposes a PCA algorithm with dynamic weighting of signals across channels. In [33], an approach based on extended Kalman filtering and SVD is proposed to extract the fetal electrocardiogram from the maternal cardiogram under conditions of arrhythmia in both the fetus and the mother. In [34], SVD is used for the decomposition of extended multichannel surface electromyography signals based on minimum MSE estimation and convolution kernel compensation. A parallel computation method for determining background noise and detecting lung rales is presented in [35]. The data matrix is factorized, and the rale detection problem is solved simultaneously with noise suppression, maintaining orthogonalization during simultaneous source separation. Recently, intelligent methods have been increasingly implemented in all noise filtering applications. For instance, in [36], deep machine learning is applied to single-channel systems to estimate the number of signal sources in the presence of noise.

Thus, the main focus of recent scientific studies on the topic under consideration is the reduction or suppression of various types of noise in MMIS. This article analyzes a method for noise level reduction and also considers the possibility of using noise in the assessment of uncertainty or measurement errors in MMIS.

Problems of Signal and Noise Separation in Big Data

A multichannel measurement information system is considered, which measures one or several physical quantities over a period of time. The experimental results are recorded in a data matrix, which in some cases can have very high dimensionality. This raises the problem: under what conditions and how can useful signals be separated from noise based on these results?

Let this data matrix $\mathbf{A}$ have dimensions $m \times n$, where *m* is the number of rows of the matrix, which in practice corresponds to the number of measurement channels, and *n* is the number of time samples, into which the realization of the random process is divided over the given time interval.

The experimental data matrix $\mathbf{A}$ can be represented as [37]

$$\mathbf{A} = \mathbf{U} \sum \mathbf{V}^T , \qquad (1)$$

where the unitary matrices $\mathbf{U}$ and $\mathbf{V}$ contain the left and right orthonormal singular vectors, respectively, such that $\mathbf{U}^T\mathbf{U} = \mathbf{I}$ and $\mathbf{V}^T\mathbf{V} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. The left singular vectors describe the basis of the row space of the matrix $\mathbf{A}$. They demonstrate a way of linearly combining the rows to obtain the principal components. The right singular vectors describe the basis of the column space of the matrix and indicate how the columns should be linearly combined to obtain the principal components. Geometrically, the left and right singular vectors determine the directions of maximum data variance in the row and column spaces, respectively. The singular vectors indicate the variables (factors) that have the greatest impact on the output parameters and their interaction patterns. These vectors can be used to construct a new data basis in which the data exhibit a simpler structure.

The singular values of the matrix $\sum$ characterize the amount of data compression along each principal component and essentially serve as an indicator of the importance of these components. They are always non-negative and arranged in descending order. Intuitively, large singular values correspond to the main features of the measured data, while small singular ones are associated with noise or less important factors. The larger the singular value, the more information it contains. By discarding components with small singular values, one can reduce the dimensionality of the data without significant loss of information. If the data contain anomalies, this manifests as unexpected singular values of the matrix $\mathbf{A}$.

The equation (1) provides a mathematical description of the SVD method for the matrix $\mathbf{A}$. If the dimensionality of the data matrix $\mathbf{A}$ is $m \times n$, then the dimensionality of the matrix $\mathbf{U}$ is $m \times m$, and matrix $\mathbf{V}$ is $n \times n$. The dimensionality of the matrix $\sum$ coincides with that of the matrix $\mathbf{A}$. If the dimensionality of the matrix $\mathbf{A}$ becomes very large, there exists a method to reduce its order without losing measurement information. Most singular values are equal to zero, and this property is used when reducing the dimensionality of the matrix $\mathbf{A}$. If the matrix $\mathbf{U}$ vectors $\vec{u}_i$ and matrix $\mathbf{V}$ vectors $\vec{v}_i$ are known, the matrix $\mathbf{A}$ will be presented in the form [37]

$$\mathbf{A} = \sum_{i=1}^{p} \sigma_i \vec{u}_i \vec{v}_i , \quad (2)$$

where p is the number of modes, and σ_i is the singular value for the i-th mode. The expression (2) in expanded form:

$$\mathbf{A} = \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2 + ... + \sigma_p \vec{u}_p \vec{v}_p = \mathbf{A}_1 + \mathbf{A}_2 + ... + \mathbf{A}_p . \quad (3)$$

In practice $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_p$, and often $\sigma_1 > \sigma_2 > ... > \sigma_p$. The first term in relation (3) usually significantly exceeds the other terms. It is determined by the basic physical processes occurring at the technical object whose parameters are being measured. The largest singular values and their corresponding vectors represent the principal modes or dominant patterns in the data. Smaller values correspond to secondary modes that capture less variance in the data. Some secondary modes may represent not errors but subtle regularities in the data. It is important to note that there is no clear boundary between principal and secondary modes.

The real data contained in the matrix $\mathbf{A}$, describe the main physical processes as well as noise (including measurement errors). The matrix can be represented as the sum of a matrix responsible for these processes and a matrix whose elements are noise. Therefore, operation (1) represents the SVD of a sum of matrices, which is generally a nonlinear operation, since such a decomposition is not a linear combination of the elements of the original matrix $\mathbf{A}$. The singular values of the matrix $\mathbf{A}$, located on the diagonal of the matrix $\sum$, are the square roots of the eigenvalues of the matrix $\mathbf{A}^T\mathbf{A}$.

The result of the SVD for the sum of matrices depends not only on the individual singular values and vectors of each matrix but also on their interaction. Computing the SVD for large matrices is a computationally expensive task, and for a sum of matrices, this complexity can increase even more. The singular values of a matrix determine its “importance” in various subspaces. When we add matrices, their singular values interact in a complex way: some may be amplified, others weakened, and new singular values may even appear. SVD defines orthogonal bases described by the singular vectors contained in the matrices $\mathbf{U}$ and $\mathbf{V}$. When matrices are added, these bases change because the new matrix has a different structure. Each singular value and its corresponding singular vectors in the SVD have a specific interpretation. For the sum of matrices, this interpretation can be more complex, as it reflects the interaction of different components. Because of adding matrices, there may also be a loss of information about the structure of the individual matrices, especially if the matrices have different ranks or their singular values differ significantly. The rank of the sum of matrices may differ from the sum of the ranks of the individual matrices, which also affects the outcome of the SVD.

In practice, decomposing a complex matrix into a sum of simpler matrices often allows one to simplify the model and improve its interpretability. However, the SVD of such a sum is not a simple linear combination of the SVDs of the individual matrices. In data analysis, SVD is often used for dimensionality reduction and for uncovering latent structures. SVD of time series makes it possible to identify various trend components. Applying SVD to the sum of matrices is useful for constructing models that take into account different types of information contained in the individual matrices.

Thus, the matrix $\mathbf{A}$ can be exactly decomposed into components according to formula (2). This formula is, in fact, an extended interpretation of formula (1). If SVD were a linear operator, then in many cases the matrix $\mathbf{A}_i \left(i = 1, 2, ..., p \right)$ would correspond directly to the influence of the i-th factor. Due to the nonlinearity of SVD, such a conclusion cannot always be made. From a technical standpoint, operations (1) and (2) describe a system where the measured data in the form of matrix $\mathbf{A}$, is input, and the output consists of the singular values σ_i which characterize the energetic properties of the decomposition modes (3), as well as the orthonormal vectors $\vec{u}_i$ and $\vec{v}_i$. The relationship between σ_i, $\vec{u}_i$, $\vec{v}_i$ and the elements of matrix $\mathbf{A}$ is nonlinear. Essentially, one needs to estimate the measured data contained in $\mathbf{A}$ based on the compressed information σ_i, $\vec{u}_i$ and $\vec{v}_i$. This problem is extremely broad. In this paper, a method is developed for partial separation of useful signals and noise in a multichannel measurement information system based on information about the singular values and orthonormal vectors of the data matrix.

The method is based on obtaining the singular modes and the hypothesis that the first mode, which significantly exceeds all other modes in terms of energy, contains information about the main physical process whose parameters are being measured. Higher-order modes describe secondary processes, measurement errors, and noise. Separating them in practice is challenging. Therefore, we first perform an analysis of the influence of these factors on the characteristics of the SVD components. The research plan includes the following stages:

1. Creating models of measurement signals in MIMS without and with noise, and forming data matrices.
2. Determination of singular values and orthogonal vectors for all data matrices.
3. Identification of patterns in the behavior of singular values and orthogonal vectors depending on measurement signal models.

4. Development of guidelines for noise estimation based on SVD and signal-noise separation methods.

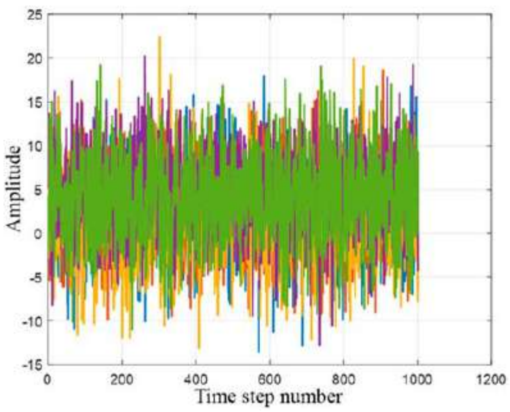
5. Experimental verification of the proposed method.

Signal models

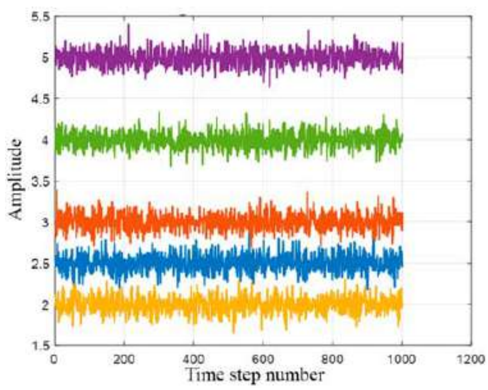
Among the different types of signals, we will first select the simplest ones, and then test the effectiveness of the proposed method on more complex models.

Case 1: Constant signals in measurement channels. There are various variants of constant signals, one of which is constant signals with the same amplitude in all channels. This is a special case, while in practice the signal amplitudes usually differ across channels. Therefore, to demonstrate the method, we choose signals with the following amplitudes in five channels: 2; 2,5; 3; 4; 5. The application of formula (1) for decomposition gives one mode with a singular value of 245,5815. If the amplitudes of the signals in the channels are the same and equal to two, then the singular value is 141,4920 and only the nature of the spatial mode distribution changes.

Case 2: Constant signals with noise. We add identical white Gaussian noise to all signals programmatically in MATLAB (Fig. 1). Since the signal amplitudes in the channels are different, the signal-to-noise ratio (SNR) q also varies across channels. In this example, the average value of q is 0,836 (the average noise amplitude is greater than the average signal amplitude).



a)



b)

Fig. 1. Distribution of instantaneous signal values in channels for SNR $q = 0,8$ (a) and $q = 41$ (b).

Unlike constant signals, decomposition (1) yields five modes with singular values shown in Fig. 2, where the index on the x-axis represents the mode number.

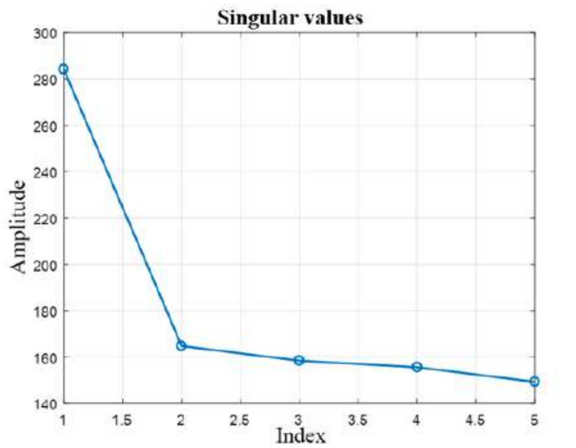
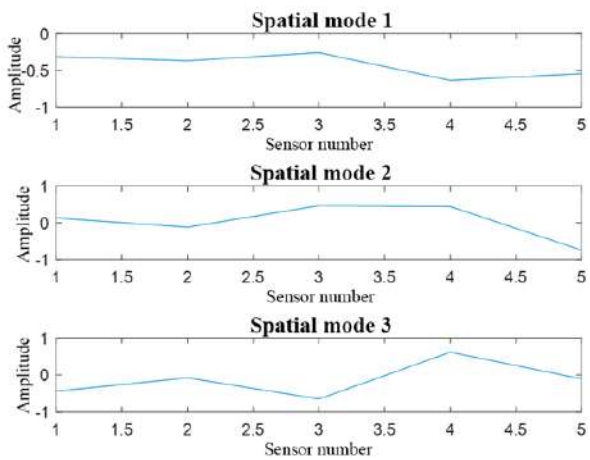
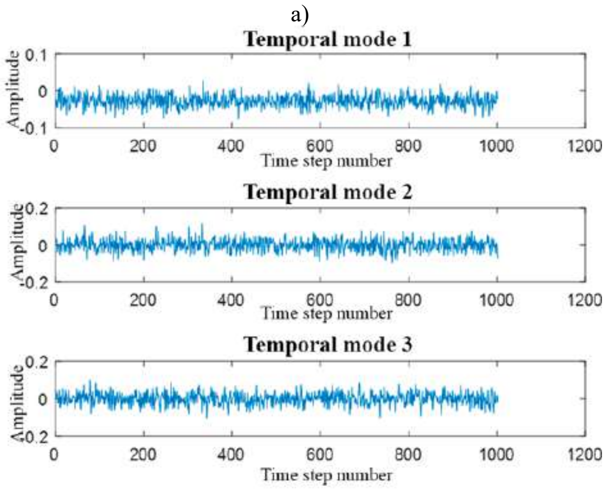


Fig. 2. Singular values of the 5 modes for signals with noise at SNR q=0,8

Examples of spatial and temporal modes, determined from the orthogonal matrices $\mathbf{U}$ and $\mathbf{V}$, are shown in Fig. 3. These modes define the spatial configuration of the matrix $\mathbf{A}$ in a multidimensional abstract space.



a)



b)

Fig. 3. Example of spatial (a) and temporal (b) modes for a 5-dimensional signal with noise.

The relative energy characteristics of the modes are shown in Fig. 4. Therefore, the first mode contains about 45% of the energy of all other modes, which are almost the same and about three times smaller than the first mode. This indicates the necessity of considering all modes.

The dependence of the largest singular value of the data matrix and the relative energy of the first mode on the signal-to-noise ratio is shown in Fig. 5. As the SNR increases, the singular values stabilize and practically do not change further. Under these conditions, the energy of the first mode increases and gradually approaches 100%.

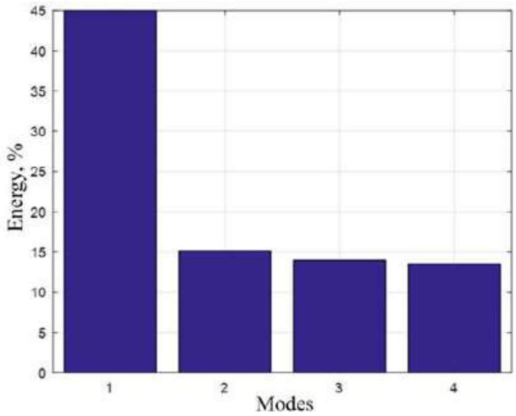
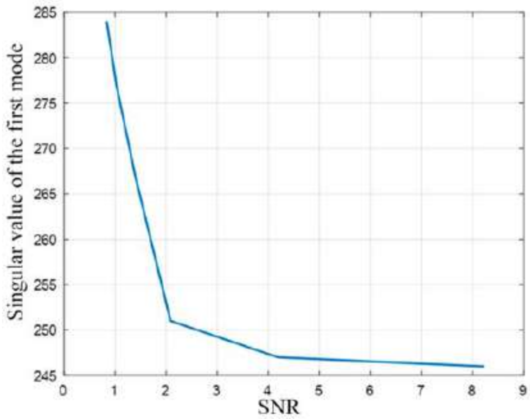
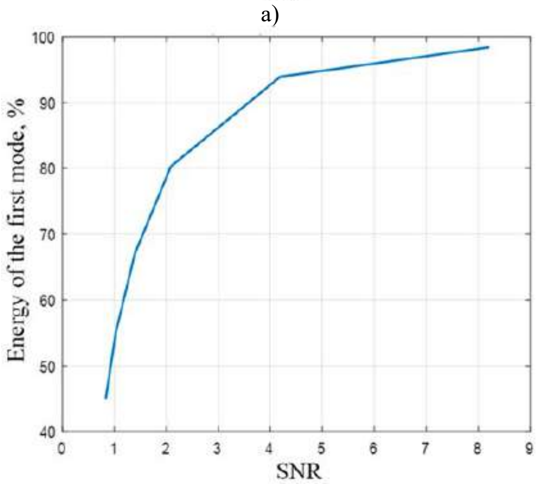


Fig. 4. Relative energy characteristics of the first modes



a)



b)

Fig. 5. Dependence of the largest singular value (a) and the relative energy of the first mode (b) on the SNR q

Partial matrices $\mathbf{A}_i \left(i = 1, 2, ..., p\right)$ describe the influence of individual factors on the measured signals. The similarity of these matrices to the data matrix $\mathbf{A}$ in relation (3) can be assessed using the cosine of the angle θ between the matrices $\mathbf{A}$ and $\mathbf{A}_i$, which is given by the formula [38]

$$\cos\theta = \frac{\langle \mathbf{A}, \mathbf{A}_i \rangle}{\|\mathbf{A}\| \|\mathbf{A}_i\|}, \tag{4}$$

where $\langle \mathbf{A}, \mathbf{A}_i \rangle$ is the scalar product of the matrices $\mathbf{A}$ and $\mathbf{A}_i$, defined by the formula: $\langle \mathbf{A}, \mathbf{A}_i \rangle = Sp\left(\mathbf{A}^T \mathbf{A}_i\right)$.

Here, Sp denotes the trace of the matrix product in parentheses, i.e., the sum of the elements on the main diagonal. The norms of the matrices $\mathbf{A}$ and $\mathbf{A}_i$ are defined similarly: $\|\mathbf{A}\| = \sqrt{\langle \mathbf{A}, \mathbf{A} \rangle}$, $\|\mathbf{A}\| = \sqrt{\langle \mathbf{A}_i, \mathbf{A}_i \rangle}$. The similarity of matrices is an analogue of their correlation and its description in terms of the cosine of the angle between the matrices is a convenient tool for multichannel systems. Figure 6 shows the dependence of $\cos\theta$ on the SNR. If the angle $\theta = 0°$, and $\cos\theta = 1$, the matrices coincide; conversely, if $\theta = 90°$ and $\cos\theta = 0$, the matrices completely lose similarity.

At low signal-to-noise ratios, the cosine of the angle between matrix $\mathbf{A}$ and the matrices $\mathbf{A}_1$, $\mathbf{A}_2$ is approximately the same. This means that under conditions of strong noise, the partial matrices $\mathbf{A}_i \left(i = 1, 2, ..., p\right)$ do not resemble the data matrix $\mathbf{A}$; therefore, their analysis may lead to incorrect conclusions about the measured data. At $q \approx 5$ or higher, the partial matrix closely resembles the data matrix $\mathbf{A}$ (the cosine of the angle θ exceeds 0,95).

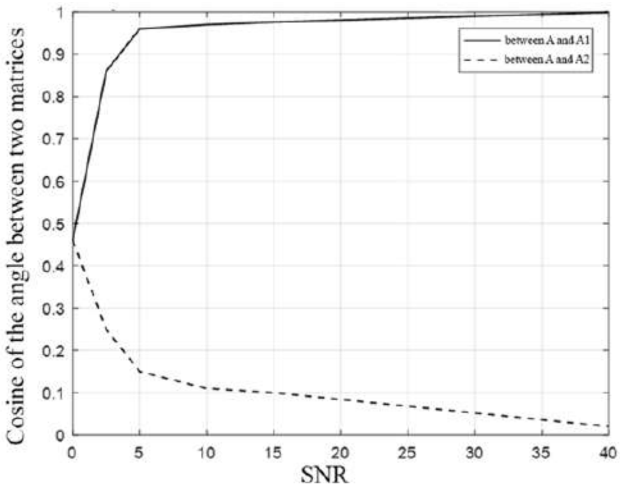


Fig. 6. Dependence of the cosine of the angle between matrices **A** and $\mathbf{A}_1$ (solid line) and between matrices **A** and $\mathbf{A}_2$ (dashed line) on the signal-to-noise ratio q

From this, it follows that many conclusions about the measurement results contained in matrix $\mathbf{A}$, can be drawn based on the analysis of matrix $\mathbf{A}_1$, which is much simpler than the full data matrix and, according to (3), is described by a single mode with singular value σ. The matrix $\mathbf{A}_1$ contains the measurement data responsible for the main physical processes occurring in the object whose parameters are being measured. The matrix $\mathbf{A}$ also contains these measurement data, but they are distorted by noise. In fact, at high signal-to-noise ratios, the useful signal is effectively filtered and cleaned of noise. However, the noise is not lost during this filtering and is captured in matrices $\mathbf{A}_i\left(i=2,3,...,p\right)$. Since under these conditions $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq ... \geq \sigma_p$, from a practical point of view, it is advisable to analyze the noise using only matrix $\mathbf{A}_2$, but only after ensuring that the experimental results were obtained under high signal-to-noise conditions. The principle of separating noise from the signal can be explained using an analogy.

The matrix $\mathbf{A}$ in a multidimensional abstract space can be mentally visualized as a vector, whose amplitude is determined by the singular values concentrated in the diagonal matrix Σ, and whose direction is defined by the orthogonal vector matrices $\mathbf{U}$ and $\mathbf{V}$. For clarity, we make a significant simplification and represent the matrix $\mathbf{A}$ as a vector $\vec{A}$ on a two-dimensional plane (Fig. 7).

Based on the previous reasoning, this vector can be approximately decomposed into a vector representing the signal, and a vector $\vec{A}_2$, representing the noise.

As can be seen from Fig. 7, the vector $\vec{A}$ is nonlinearly related to the vectors $\vec{A}_1$ and $\vec{A}_2$.

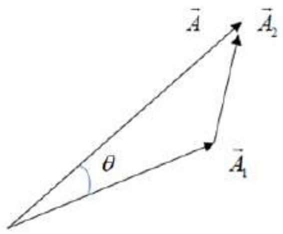


Fig. 7. Simplified illustration of the principle of separating noise from the signal

It corresponds to the geometric sum of these vectors, which is a simplified geometric analogy of relations (2) and (3) under the condition.

At high signal-to-noise ratios, $\cos\theta \to 1$, and the angle itself $\theta \to 0°$ (Fig. 7). As soon as the angle θ approaches zero, the vectors $\vec{A}_1$ and $\vec{A}_2$ become close to the vector $\vec{A}$, meaning that instead of the nonlinear operation of forming the geometric sum $\vec{A} = \vec{A}_1 + \vec{A}_2$ we effectively get an arithmetic sum $\mathbf{A} \approx \mathbf{A}_1 + \mathbf{A}_2$, which can be performed using a linear operation. From this, we conclude that the signal and noise behave additively. The measurement results are contained in a matrix $\mathbf{A}$, which after transformation (1)…(3), go to the matrices $\vec{A}_1$ and $\vec{A}_2$. Analyzing the matrix $\mathbf{A}_2$ allows us to obtain information about noise, which also includes measurement errors. If external and internal noise is significantly reduced during experiments, analyzing the matrix $\mathbf{A}_2$ makes it possible to estimate measurement errors, which are often difficult to distinguish from noise. The remaining challenge is separating noise from the signal at low signal-to-noise ratios.

Case 3: signals with noise varying across channels. Now consider an example with a different data matrix: instead of constant signals in the measurement channels, there are harmonic signals with varying amplitudes and frequencies, as well as signals with linear and nonlinear amplitude modulation, to which noise is added (Fig. 8).

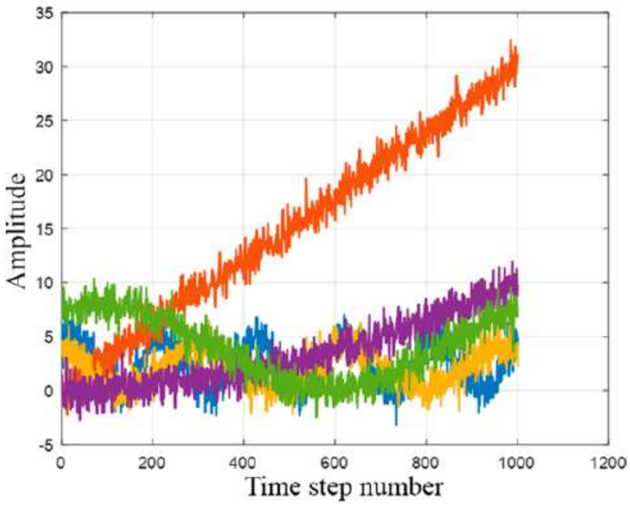


Fig. 8. Realizations of random signals with noise in five channels at q=4,18

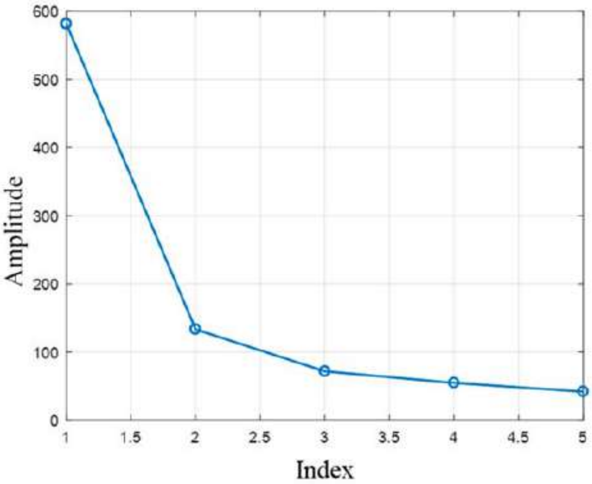
The average signal-to-noise ratio by amplitude was 4,18.

The distribution of singular values and the relative energy of the modes are shown in Fig. 9.

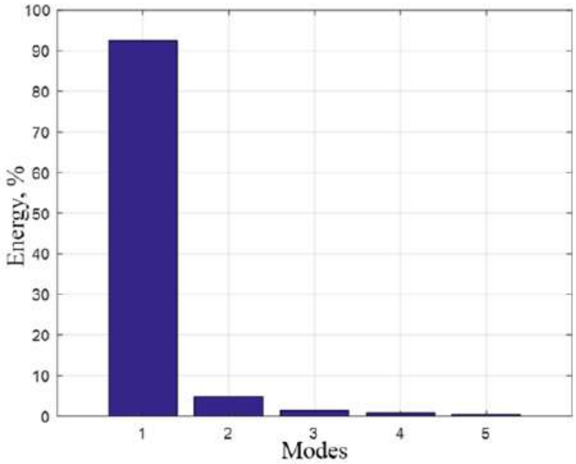
At this SNR, the cosines of the angles between the matrices $\mathbf{A}$, $\mathbf{A}_1$ and between $\mathbf{A}$, $\mathbf{A}_2$ are 0,9616 and 0,2202, respectively. These values roughly coincide with the corresponding values for constant signals. Similar results are observed for other types of signals at comparable signal-to-noise ratios. Therefore, the similarity of the main data matrix $\mathbf{A}$ with the partial matrices $\mathbf{A}_i$ primarily depends on the energy characteristics of the modes (i.e. SNR) rather than on the specific shape of the signals.

Case 4: Analysis of Experimental Studies. Consider the results of two experimental studies conducted by the authors.

The first experiment used a measuring complex containing four digital strain gauges to measure the deformation of a mechanical installation. Four realizations of a non-stationary random process were obtained (Fig. 10).



a)



b)

Fig. 9. Distribution of singular values (a) and relative modal energies of the data matrix (b) for the 3rd case

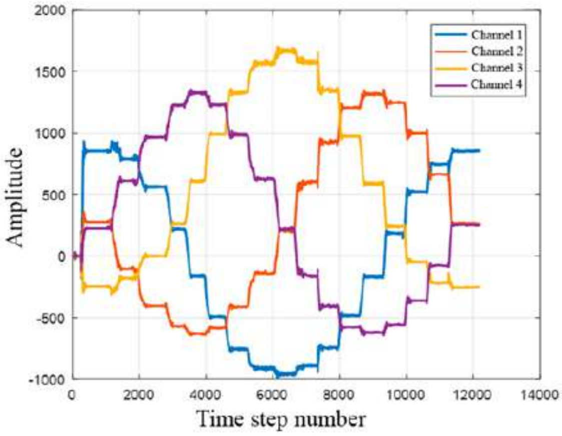
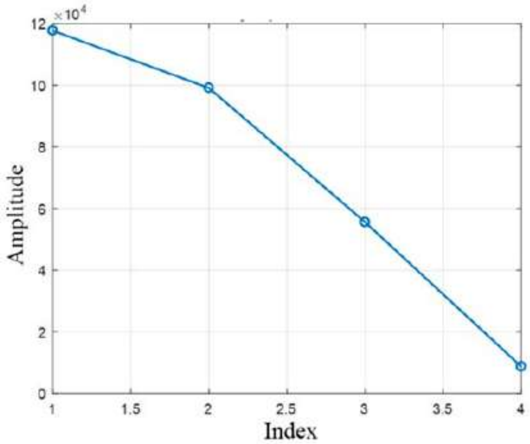


Fig. 10. Time dependencies of the deformations of the mechanical setup

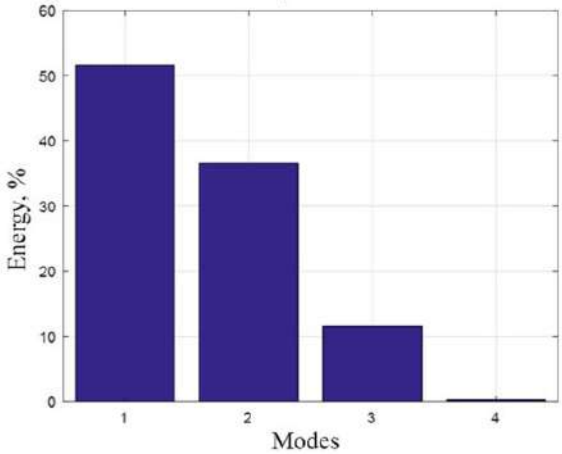
The cosines of the angles between the data matrix $\mathbf{A}$ and the partial matrices $\mathbf{A}_i$ according to formula (4), are:

- Between $\mathbf{A}$ and $\mathbf{A}_1$: 0,7185.
- Between $\mathbf{A}$ and $\mathbf{A}_2$: 0,6050.
- Between $\mathbf{A}$ and $\mathbf{A}_3$: 0,3391.
- Between $\mathbf{A}$ and $\mathbf{A}_4$: ,.0531.
- Between $\mathbf{A}_1$ and $\mathbf{A}_2$: $1{,}4664 \cdot 10^{-19}$

The singular values of the modes and their relative energy are shown in Fig. 11.



a)



b)

Fig. 11. Singular values of the modes (a) and their relative energy (b) in the first experiment.

This means that the similarity between the data matrix $\mathbf{A}$ and $\mathbf{A}_i$ is preserved practically up to the third partial matrix, while there is no correlation between the partial matrices themselves. The cosine of the angle between matrices $\mathbf{A}$ and $\mathbf{A}_1$ is particularly significant. In the given experimental study, it equals 0,7185, indicating that one cannot assert a strong similarity between these matrices, and the partial matrices cannot be attributed with noise properties, since their correlation with the main matrix remains high (0,605). All of this reflects the nonlinearity of the SVD transformation.

In the second experiment, stationary pressure processes were studied (Fig. 12).

The realizations of this process are described by a single mode. The cosine of the angle between matrices $\mathbf{A}$ and $\mathbf{A}_1$ is very close to 1, while between matrices $\mathbf{A}$ and $\mathbf{A}_2$ it is 0,004. The time dependencies of pressure for matrix $\mathbf{A}_1$ practically replicate the previous figure. This indicates that matrix $\mathbf{A}_1$ represents the main physical processes occurring in the setup. Since the similarity between matrices $\mathbf{A}$ and $\mathbf{A}_2$ is practically absent, matrix $\mathbf{A}_2$ can be considered to represent noise (Fig. 13).

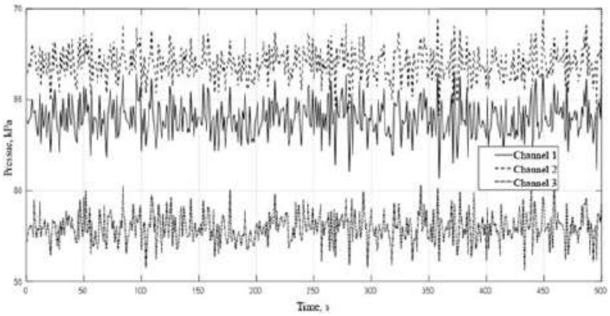


Fig. 12. Time dependences of pressure in three channels of the measuring information system

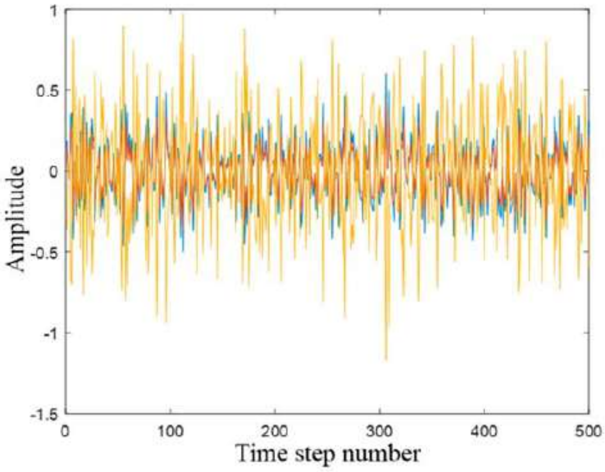


Fig. 13. Instantaneous noise values for the second experiment

Analysis of secondary modes in singular decomposition of a data matrix

Therefore, the analysis method using SVD allows us to identify the main modes in the experimental data, in which a significant part of the energy and information is concentrated. The main modes are associated with essential patterns in the data that likely represent real phenomena under study. However, there are also non-primary (additional, secondary) modes that are usually not analyzed in practice.

Can we assume that secondary modes may describe measurement errors during data collection? There are no theorems that definitively answer this question. In many cases, secondary modes with significantly smaller singular values may characterize noise or measurement errors. The assumption is that the main signal in the data has a larger variance than the noise, and measurement errors are smaller in magnitude than the main components of the signal. However, it is not always straightforward to determine the boundary between signal and noise, or between noise and random errors. It should also be noted that some secondary modes may occasionally represent subtle patterns in the measurement data. To reduce the impact of SVD nonlinearity, it is necessary to ensure a high value of $\cos\theta \to 1$. For the first experiment, this condition is not met, whereas for the second experiment, it is fully satisfied. This is due to the non-stationarity of the random processes in the first experiment. As a result, it is impossible to determine the noise in the first experiment using the proposed method, since it contains components of the main mode due to SVD nonlinearity. The second experiment, in contrast, is described by a single mode and is characterized by weak correlation between the data matrix and partial matrices with indices 2 and higher. At the same time, the similarity between matrices $\mathbf{A}$ and $\mathbf{A}_1$ is very high. Thus, for separating signal and noise, the described method can be applied provided that $\cos\theta$ between matrices $\mathbf{A}$ and $\mathbf{A}_1$ or the relative energy of the first mode exceeds a certain threshold, for example, 0,9 or 90%, respectively. The lower this threshold, the larger the errors in noise determination. To evaluate these errors, we obtain the matrix $\mathbf{A}_2$ at different signal-to-noise ratios. The matrix $\mathbf{A}_2$ at SNR $q \approx 41$ for the second case considered in the article is taken as the reference, i.e., $\mathbf{A}_2 = \mathbf{A}_{ref}$. For lower SNR q, new matrices are calculated and compared with the reference matrix. When SNR q decreases, some real important signal characteristics may be misclassified as noise if they make a small contribution to the total variance. Thus, the data matrix $\mathbf{A}$ contains both signals and noise; the first-order partial matrix $\mathbf{A}_1$ mainly represents signals, while all higher-order partial matrices primarily represent noise, with the highest level occurring in matrix $\mathbf{A}_2$. If a low level of internal noise in the measurement system is ensured and external noise is absent, matrix $\mathbf{A}_2$ will contain information about random measurement errors or measurement uncertainty. Most systematic measurement errors are difficult to determine with the proposed method, as they may appear in the first-order matrix $\mathbf{A}_1$. To enable the separation of signals and noise, an appropriate signal-to-noise ratio must be maintained. The reference matrix $\mathbf{A}_{ref}$ is illustrated by the dependencies shown in Fig. 14.

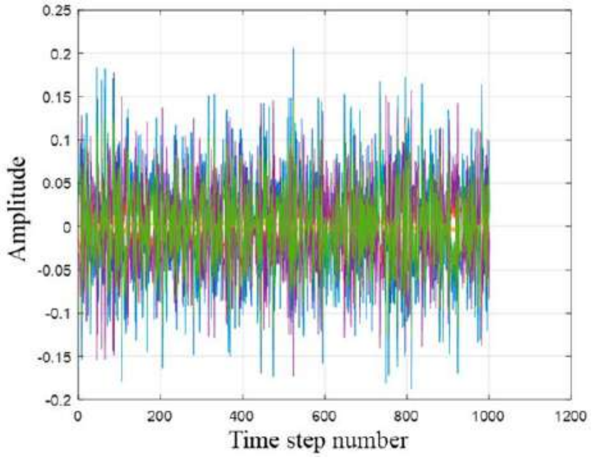


Fig 14. Time dependencies of the parameters of the reference matrix $\mathbf{A}_{ref}$ at a signal-to-noise ratio of 41

The time dependences of the matrix $\mathbf{A}_2$ parameters at other values of q have a similar form. It is advisable to consider the ordinates on all graphs, as they characterize the noise amplitudes. If these amplitudes are large, it indicates that energy is flowing from the first mode to higher-order modes due to nonlinear effects in the SVD process. It is not the amplitude of the noise that matters, but the signal-to-noise ratio.

Fig. 15 shows the dependence of the relative error in noise estimation on q. The relative errors were calculated as the ratio of the maximum value of the matrix $\mathbf{A}_2$ at the corresponding q to the maximum value of the data matrix $\mathbf{A}$.

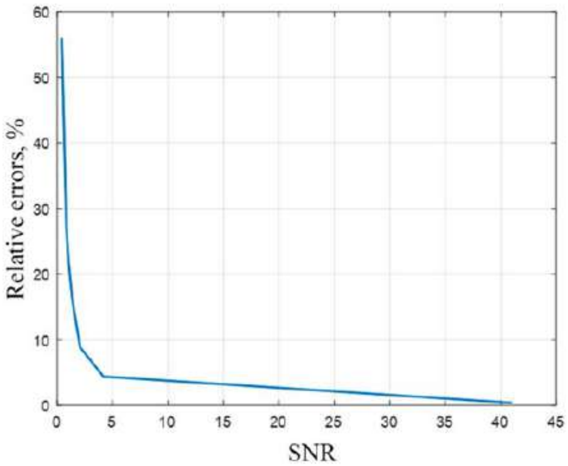


Fig. 15. Dependence of the relative errors in noise estimation on the signal-to-noise ratio

Figure 15 shows that for any data matrix, it is possible to determine conditions under which noise together with measurement errors will not exceed a specified level. Separation of noise and measurement errors in the matrix $\mathbf{A}_2$ can be done by machine learning methods or other methods if there are any differences between them.

It should be noted that SVD is a popular method for data dimensionality reduction. However, there are other methods, each with its advantages and disadvantages. First, there is the Principal Component Analysis (PCA) [39]. The first few principal components of PCA, corresponding to the largest eigenvalues of the covariance matrix, coincide with the first few left singular vectors of SVD. However, SVD is a more general method and can be applied to arbitrary matrices. Second, there is Linear Discriminant Analysis (LDA) [40], which seeks linear combinations of the original variables that maximize the separation of different data classes. LDA is effective for classification tasks when classes are clearly separated but is less effective for small sample sizes. Third, there is t-Distributed Stochastic Neighbor Embedding [41], a nonlinear method that preserves the local structure of data in a low-dimensional space. This method is often used for visualizing high-dimensional data, but it can be quite slow for large datasets, and the results may depend on the initial initialization.

Fourth, there are machine learning methods [42] based on neural networks. A hidden layer with fewer neurons performs the dimensionality reduction. Neural network–based methods require large amounts of training data and are prone to overfitting. Each of these methods has its strengths and weaknesses. The choice of method depends on the specific task and requires analysis of the data and research objectives. In some cases, using a combination of methods may be advisable to achieve acceptable results.

To determine the role of each partial matrix in forming noise and measurement errors, a model was constructed in the form of parallel-connected MMIS and a reference measurement information system (RMIS), where measurement errors are approximated as zero. Partial matrices for MMIS are denoted as $\mathbf{A}_i$, and for RMIS as $\mathbf{A}_{in}$. The similarity between the data matrices $\mathbf{A}_1$ and $\mathbf{A}_{1n}$ is already close to one, allowing the analysis of random processes that determine the physical properties of the system under different distributions of noise and measurement errors. The results are presented for a signal-to-noise ratio of 14. The similarity (measured via cosine of the angle) between the matrices $\mathbf{A}_2$ and $\mathbf{A}_{2n}$, $\mathbf{A}_3$ and $\mathbf{A}_{3n}$, $\mathbf{A}_4$ and $\mathbf{A}_{4n}$, $\mathbf{A}_5$ and $\mathbf{A}_{5n}$ ranges from 0,6976 to 0,7669. Cross-similarity, e.g., between $\mathbf{A}_2$ and $\mathbf{A}_3$, $\mathbf{A}_{2n}$ and $\mathbf{A}_{3n}$ is close to zero. The study is conducted for the case where the level of random errors and noise is approximately equal.

For comparable noise and measurement errors, we consider simulation results with different distributions: noise distributed uniformly, and measurement errors normally. The mixture of noise and errors already deviates from a normal distribution. After singular value decomposition, we obtain practically identical matrices $\mathbf{A}_1$ for MMIS and $\mathbf{A}_{1n}$ for RMIS, with similarity close to 1, even at a relatively low signal-to-noise ratio (Fig. 16).

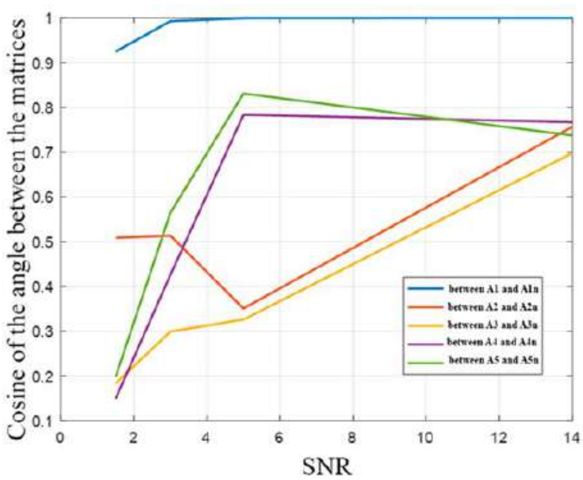


Fig. 16. Dependence of the similarity of different matrix pairs on the signal-to-noise ratio

The similarity between the matrices $\mathbf{A}_2$ and $\mathbf{A}_{2n}$,

$\mathbf{A}_3$ and $\mathbf{A}_{3n}$, $\mathbf{A}_4$ and $\mathbf{A}_{4n}$, $\mathbf{A}_5$ and $\mathbf{A}_{5n}$ at a signal-to-noise ratio $q=14$ is in the range from 0,7 to 0,77 (Fig. 16) and increases at higher q. Therefore, at high SNR, the analysis of noise and measurement errors can be performed using MMIS data alone, without the use of RMIS. At low q, the matrices $\mathbf{A}_i$ for MMIS and RMIS differ, and the similarity between the input data matrix and the noise matrices is low.

It is important to clarify the physical meaning of the noise matrices $\mathbf{A}_i$, which describe the noise components in MMIS. Since the input data were generated through modeling, the noise and measurement errors are known, and the similarity of matrices $\mathbf{A}_i$ with the actual noise can be determined. This similarity is small (approximately 0,3) and increases only slightly with increasing q. The similarity of these matrices with measurement errors may exceed 0,4 and decreases as the SNR grows. The similarity of matrices $\mathbf{A}_{in}$ for RMIS with noise is much higher and can exceed 0,5; it may increase with increasing q for $\mathbf{A}_{5n}$, and decrease for all other $\mathbf{A}_{in}$. Their similarity with measurement errors is approximately zero, as expected, since measurement errors were not introduced in RMIS during modeling. This analysis corresponds to the case of measurement errors comparable to noise.

For large noise (a third of the average signal amplitude) and small measurement errors (10 times less noise), in MMIS the similarity of noise matrices with actual noise is in the range 0,44–0,47, and in RMIS 0,42–0,47. The similarity of the noise matrices with measurement errors is close to zero for SNR of 3.

For small noise, 10 times smaller than the measurement errors, at a signal-to-noise ratio of 27, the similarity of MMIS noise matrices with noise is 0,05–0.08, and with measurement errors 0,42–0,46; for RMIS, the similarity with noise is about 0,42–0,45 and near zero with measurement errors, since no measurement errors were assumed in RMIS.

Thus, the analysis of partial matrices from the singular value decomposition of the input data matrix shows the impossibility of clearly assigning these matrices either to noise or to measurement errors. The SVD method does not allow determining the responsibility of the *i*-th partial matrix $(i = 2,\dots,5)$ for forming noise or measurement errors, which in general cannot be separated using this method. The most favorable case occurs at high q, when the singular values of the first matrix $\mathbf{A}_1$ are much larger than those of the other partial matrices $\mathbf{A}_i$ for $i=3,\dots,5$. In this case, the matrix characterizes the intensity of measurement and noise errors in the MMIS channels. Thus, SVD allows to reduce the noise level, and the analysis of physical processes can be carried out using the first partial matrix $\mathbf{A}_1$ instead of the traditional data matrix $\mathbf{A}$.

The main applications of this method include:

1. Noise filtering. Using only the large singular values to reconstruct the data matrix allows the removal of noise and the extraction of a “cleaned” signal.
2. Estimation of measurement errors. Analysis of small singular values and corresponding partial matrices allows us to estimate the contribution of hardware errors and external interference.
3. Diagnostics of the information-measurement system. Large values of the noise component may indicate sensor malfunctions or calibration problems.

Conclusions

The problem of noise filtering has long been a topic of significant interest. The article considers a method for separating noise from signals in a multi-channel measurement information system with the aim of further possible determination of measurement errors that may be similar to noise. It is shown that using singular value decomposition provides a systematic approach to separating signal and noise at a sufficiently high signal-to-noise ratio, although complete separation is not achievable.

In the proposed method, after performing SVD, an analysis of the partial matrices is conducted, whose sum equals the data matrix obtained from multichannel measurements. The data matrix contains both useful signals and noise. It is established that these can be separated if the cosine of the angle between the data matrix and the first partial matrix approaches 1, while the cosine of the angle between the data matrix and the second partial matrix approaches 0. Such conditions are achieved when the signal-to-noise ratio exceeds a certain threshold.

The first partial matrix contains useful signals, while higher-order matrices contain noise. Typically, the noise level in the second partial matrix is much higher than in higher-order matrices. At a low level of internal noise in the multichannel measurement information system, and with external noise removed, the second partial matrix primarily contains random measurement errors. Systematic measurement errors cannot always be determined by the proposed method, as they often enter the first partial matrix along with the useful signals.

The main applications of the proposed method are noise filtering, assessment of measurement errors, and diagnostics of the state of the measurement information system.

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Метод оцінювання шумів в багатоканальній вимірювальній інформаційній системі на основі декомпозиції сингулярних значень матриці даних

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Анотація

Фільтрація шумів широко впроваджена в теорії і техніці обробки сигналів. Значно менша кількість наукових робіт присвячена вилученню шумів з реалізацій випадкових процесів з метою їх аналізу для специфічних завдань. У статті запропоновано метод розділення сигналів і шумів у багатоканальній вимірювальній інформаційній системі. Для цього використовується матриця експериментальних даних і за допомогою декомпозиції сингулярних значень (singular value decomposition – SVD) здійснюється аналіз сингулярних мод цієї матриці та парціальних матриць, які є складовими матриці даних. Визначені умови, при яких парціальна матриця першого порядку буде описувати сигнали в каналах системи, а матриці вищих порядків містять шумові компоненти. Для цього косинус кута між матрицею даних та першою парціальною матрицею повинен наближатися до одиниці, а між матрицею даних та другою матрицею – до нуля. Такі умови досягаються у випадках перевищення порогового рівня відношенням сигнал/шум. Отримані шуми в окремих випадках можуть використовуватися для визначення похибок вимірювання.

Ключові слова: багатоканальна вимірювальна інформаційна система, декомпозиція сингулярних значень, матриця експериментальних даних, фільтрація шуму.