UDC 53.084 DOI: 10.30837/2663-9564.2025.2.01

DETERMINATION OF EQUIVALENT CIRCUIT PARAMETERS OF RESISTORS AND CAPACITORS FROM MEASUREMENTS OF THEIR IMPEDANCE FREQUENCY CHARACTERISTICS

Zygmunt L. Warsza[1], Jacek Puchalski[2]

[1]*Polish Metrological Society, Warszawa, Poland*
[2]*Central Office of Measures GUM, Warszawa, Poland*

Abstract
The paper presents a method for determining parameters of equivalent circuits of resistors and capacitors as connection of two-terminal elements with single linear resistance, inductance or capacitances. The values and uncertainties of these parameters are estimated using the least squares method for measurements of the frequency characteristic of the module of impedance. This task is mathematically complicated, because usually you get a system of nonlinear equations, which is not analytically solvable. To obtain linear equations, it was proposed to use the method by changing variables. This method was previously developed by authors for the regression of nonlinear functions and has already been successfully used in metrological tasks.
Key words: equivalent circuits, resistance, capacitance, inductance, frequency characteristic, module of impedance.

1. Introduction

In the design and manufacture of electronic circuits, it is necessary to know the basic parameters of the equivalent circuits of the components used in them, including resistors, capacitors and inductors. Determining these parameters on the basis of measurements of frequency characteristics of passive components, even for their simplified equivalent circuits with ideal resistances, capacitances and inductances, is a complicated task. A system of nonlinear equations is obtained, which usually has no analytical solutions. In the literature on the analysis of equivalent schemes of these elements, no methods for estimating the values and uncertainties of indirect measurement results described by nonlinear functions have been used, e.g. [2-6].

Equivalent diagrams of a resistor and a capacitor with 5 ideal linear single-parameters as resistance R, inductance L and capacitance C were considered by Kubis and Warsza in papers [7-13]. From measurements of the modulus of impedance of these schemes at several frequencies, they determined values of their 5 parameters using the numerical Monte Carlo method, but without assessing uncertainty.

To determine the values and uncertainties of the parameters of the equivalent schemes of passive elements, we propose the use of a method with linearization of functions by changing their variables. It simplifies the process of metrological analysis of systems. The authors presented and discussed this method at several applications in metrology and measurement technology at previous national conferences PPM and MKM and international conferences MathMet, AMCTM XII and at the IMEKO Congress, and also published [14-22]. This method will be used below to determine the values and to evaluate the accuracy of the linear parameters of the simplified equivalent scheme of the resistor with only three parameters R, L, C and the square impedance components of the capacitor with two capacitances and three resistances.

The examples will use the results of measurements of the impedance module of both passive components for $n = 10$ frequencies. For the resistor, the frequency responses, nominal and adjusted to the parameters of the system (WTLS) with an uncertainty corridor and three parameters of the schematic will be determined. For the capacitor, the resistive and reactance components of its impedance are matched, and their uncertainties are determined by the law of propagation LPU.

2. Description of the variable-change linearization method

In the linearization method described below, for a nonlinear function $y = f(x)$ a linear equation is created in new Cartesian coordinates ξ, ψ, which takes the form of

$$\psi(y, \boldsymbol{\beta}) = \theta_1 \xi(x, \boldsymbol{\beta}) + \theta_0. \quad (1)$$

After changing the coordinates x, y on ξ, ψ can be adjusted the parameters of the equation (1) using the linear regression (1) to the measurement data of the tested points according to the criterion WTLS least squares. All parameters of the fitted curve are given by the vector $\boldsymbol{p} = [\theta_1, \theta_0, \boldsymbol{\beta}]^T$. A criterion function is specified by errors in the new coordinates denoted by n-dimensional vectors $\boldsymbol{\Delta\xi}$ and $\boldsymbol{\Delta\psi}$ by the covariance matrix $\boldsymbol{U}$ size of $2n$ x $2n$:

$$\phi_{\xi\psi} = \begin{bmatrix} \boldsymbol{\Delta\xi} \\ \boldsymbol{\Delta\psi} \end{bmatrix}^{\boldsymbol{T}} \boldsymbol{U}^{-1} \begin{bmatrix} \boldsymbol{\Delta\xi} \\ \boldsymbol{\Delta\psi} \end{bmatrix}, \quad (1a)$$

where: $\boldsymbol{U}$ is the symmetric covariance matrix for the new coordinates ξ and ψ.

This matrix is formed by both sides' multiplication of the covariance matrix for x and y by diagonal matrices of first n elements as derived values $\xi'(x,\boldsymbol{\beta})$ and about the next n elements $\psi'(y,\boldsymbol{\beta})$. Inverse covariance matrix $\boldsymbol{U^{-1}}$ denoted by elements of this quadratic matrixes $\boldsymbol{U_{11}}$, $\boldsymbol{U_{12}}$ i $\boldsymbol{U_{22}}$,size of n x n i.e.

$$\boldsymbol{U^{-1}} = \begin{bmatrix} \boldsymbol{U_{11}} & \boldsymbol{U_{12}} \\ \boldsymbol{U_{12}^T} & \boldsymbol{U_{22}} \end{bmatrix}. \quad (1b)$$

A minimum search is a state in new coordinates that meets the following conditions:

$$\boldsymbol{\nabla}_{\boldsymbol{\xi_p}}\phi_{\xi\psi} = \frac{\partial\phi_{\xi\psi}}{\partial\Delta\xi} = \boldsymbol{0}, \frac{\partial\phi_{\xi\psi}}{\partial\theta_1} = 0 \text{ and } \frac{\partial\phi_{\xi\psi}}{\partial\theta_0} = 0. \quad (2a, b, c)$$

The first of these conditions only is analytically solvable. The local minimum of the inverse of the effective covariance matrix can be obtained for:

$$\boldsymbol{U_{Yeff}^{-1}} = \boldsymbol{U_{22}} - \left(\boldsymbol{U_{12}^T} + a\boldsymbol{U_{22}}\right)\boldsymbol{T^{-1}}(\boldsymbol{U_{12}} + a\boldsymbol{U_{22}}), \quad (3)$$

where $\boldsymbol{T} = \boldsymbol{U_{11}} + \theta_1\left(\boldsymbol{U_{12}^T} + \boldsymbol{U_{12}}\right) + {\theta_1}^2\boldsymbol{U_{22}}$.

Effective inverse covariance matrix $\boldsymbol{U_{Yeff}^{-1}}$ is diagonal when it is assumed that correlations occur only between coordinates at measurement points with a correlation coefficient ρ. Then the diagonal elements of $\boldsymbol{U_{Yeff}}$ specifying the variance (the square of the effective uncertainty) are given by the expression:

$$u_{eff}^2 = \theta_1^2 u^2(\xi) - 2\theta_1\,\rho u(\xi)u(\psi) + u^2(\psi), \quad (4)$$

where $u(\xi) = |\xi'(x,\boldsymbol{\beta})|u(x);\ u(\psi) = |\psi'(y,\boldsymbol{\beta})|u(y)$.

The criterion function is quasi-quadratic, i.e.:

$$\phi_{\psi\xi}(\theta_1) = \theta_1^2\left(S_{\xi\xi} - \frac{S_\xi^2}{S}\right) + 2\left(\frac{S_\xi S_\psi}{S} - S_{\xi\psi}\right)\theta_1 + S_{\psi\psi} - \frac{S_\psi^2}{S}, \quad (5)$$

where: $S = \boldsymbol{1^T}\,\boldsymbol{U_{eff}^{-1}}\boldsymbol{1} = \sum_{i=1}^{n}\sum_{j=1}^{n}[\,u_{Yeff}^{-1}]_{ij} > 0$,

$S_\xi = \boldsymbol{\xi^T}\boldsymbol{U_{Yeff}^{-1}}\boldsymbol{1} = \boldsymbol{1^T}\boldsymbol{U_{Yeff}^{-1}}\boldsymbol{\xi}$, $\quad S_{\xi\xi} = \boldsymbol{\xi^T}\boldsymbol{U_{Yeff}^{-1}}\boldsymbol{\xi}$,

$S_\psi = \boldsymbol{\Psi^T}\boldsymbol{U_{Yeff}^{-1}}\boldsymbol{1} = \boldsymbol{1^T}\boldsymbol{U_{Yeff}^{-1}}\boldsymbol{\Psi}$, $\quad S_{\psi\psi} = \boldsymbol{\Psi^T}\boldsymbol{U_{Yeff}^{-1}}\,\boldsymbol{\Psi}$,

and $\theta_0 = (\,S_\psi - \theta_1\,S_\xi)/S$.

Vectors $\boldsymbol{\xi}$, $\boldsymbol{\Psi}$ about the size n x 1 are determined by the coordinates of the measurement points $\boldsymbol{X} = [x_1, \dots x_n]^{\boldsymbol{T}}$, $\boldsymbol{Y} = [y_1, \dots, y_n]^{\boldsymbol{T}}$ through transformations of functions $\xi(x,\boldsymbol{\beta})$, $\psi(y,\boldsymbol{\beta})$ with initial input parameters $\boldsymbol{\beta} = \boldsymbol{\beta_0}$. It is also assumed that random variables x and y are not correlated. In cases where the $\boldsymbol{\beta}$ is a one-dimensional vector with the value β, a two-dimensional criterion function is obtained. A typical chart of this chart is shown in Fig. 1.

A general flowchart for the determination of the standard and extended uncertainty for the parameterized curve is given in Fig. 2. In the first phase, the parameters of the curve are adjusted using the least squares method, assuming that both the coordinates of the measurement points and the covariance matrix are known, in the second phase, by numerical differentiation of the curve parameters, the covariance matrix of the parameters is determined.

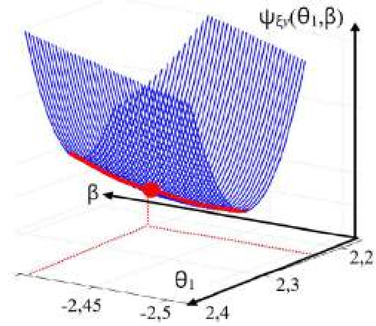


Fig. 1. Graph of typical criterion function in two-dimensional input area θ_1, β

From the analytical derivatives with respect to the parameters of the parameterized curve, the coverage interval at each point of the adjusted curve is obtained.

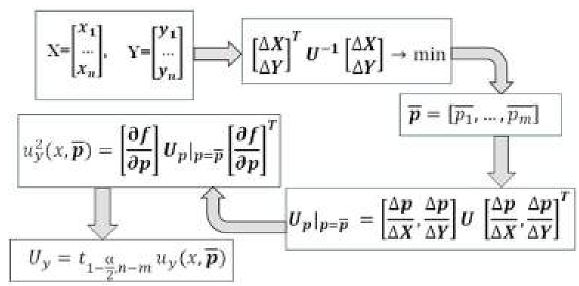


Fig. 2. Scheme for determining the least squares fit uncertainty corridor

This diagram shows that it is possible to determine the standard and expanded uncertainty at any point in the curve to be fitted. Uncertainty of all m-parameters $\boldsymbol{p} = [p_1, \dots, p_m]^{\boldsymbol{T}}$ shall be estimated according to the *Law of Propagation of Uncertainty* (LPU), as the product of the input covariance matrix (generally of the size $2n$ x $2n$) $\boldsymbol{U_{in}}$ is multiplied by both sides of the matrix $\boldsymbol{C}$ including the sensitivity coefficients (generally of the size m x $2n$):

$$\boldsymbol{U_P} = \boldsymbol{C}\boldsymbol{U_{in}}\boldsymbol{C^T}. \quad (6)$$

The sensitivity coefficients, as the first derivatives, are calculated by numerical differentiation of each parameter $p_i\,(i = 1, \dots, m)$ according to all input quantities (generally there are $2n$ differences for each measurement point $x_i\,, y_i$). The first derivatives are estimated from the formula:

$$C_{ij} = \frac{\partial p_i}{\partial z_j} \approx \frac{p_i(z_j + \Delta z_j) - p_i(z_j - \Delta z_j)}{2\Delta z_j}, \quad (7)$$

where: $i = 1, \dots, m$, $j = 1, \dots, 2n$, whereas z_j is one of the coordinates of the measurement $(x_j\,, y_j)$.

The increment values are selected to meet the requirements for assessing the values of the first derivatives with respect to Δz_j for $\Delta z_j \ll z_j$. When all parameters are directly the actual parameters of the curve to be fitted, then the standard and extended uncertainty is obtained from the LPU:

$$u_y^2(x) = \boldsymbol{SU_pS^T} \text{ and } U = t_{1-\frac{\alpha}{2},n-m}u_y, \quad (8a,b)$$

where $\boldsymbol{S} = [\frac{\partial y}{\partial p_1}, \dots, \frac{\partial y}{\partial p_m}]$ is a vector with a size of $1 \text{ x } m$ containing the sensitivity coefficients.

The sensitivity coefficients are determined analytically as the first partial derivatives with respect to the parameters of the system with matched values and for selected values *x*.

The diagram in fig. 2 does not include the method of changing variables used in the analyzed examples, described above. This method simplifies the adjustment of the parameters of the curve described by the nonlinear function, because in the new variables, when the least squares criterion is used, the criterion function is minimized.

The equation describing the curve in the new variables is equivalent to the equation in the original coordinates x and y. The values of a criterion function in the new variables are close to the original values of this function if both new variables are dependent individually on the original variables according to formula (1). If each of the new variables depends on both primitive coordinates, then the new values will be different from those for the primitive variables.

3. Parameters of the resistor and their uncertainties

The subject of the research will be parameters of the real resistor equivalent circuit shown as diagram in fig. 3. This circuit consists of two parallel branches. In the upper one there is a resistance R and two equal inductances L connected in series with it, each with an imaginary component of the impedance $j\omega L$. The impedance of this branch is $2j\omega L + R$ and it is connected in parallel with a capacitor with capacitive reactance $1/j\omega C$.

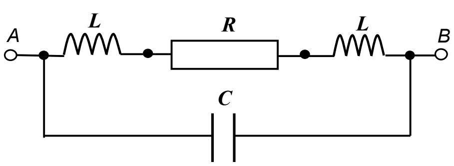


Fig. 3. Equivalent diagram of the resistor under test

The complex impedance $\boldsymbol{Z}$ the two terminals A B circuit replacing the actual resistor is:

$$\boldsymbol{Z} = \frac{1}{j\omega C + \frac{1}{2j\omega L + R}} = \frac{2j\omega L + R}{1 - 2\,\omega^2 LC + j\omega}. \quad (9)$$

Square of the module $|\boldsymbol{Z}|$ of impedance $\boldsymbol{Z}$ is described by the expression

$$|\boldsymbol{Z}|^2 = \frac{4\omega^2 L^2 + R^2}{(1 - 2\,\omega^2 LC)^2 + \omega^2 R^2 C^2}. \quad (10)$$

The following transformations will be performed: multiplying both sides of the equation (10) by the denominator of the left side of equation (9), transferring the term $|\boldsymbol{Z}|^2\omega^2 R^2 C^2$ to the right side and dividing both sides, by the 1-$|\boldsymbol{Z}|^2\omega^2 C^2$. This gives a linear equation in the new coordinates of the form:

$$\psi = \Theta_1 \xi + \Theta_0, \quad (11)$$

where: $\Theta_0 = R^2$, $\Theta_1 = L$, and for $1 - |\boldsymbol{Z}|^2\omega^2 C^2 \neq 0$ new variables ψ, ξ are defined as follows:

$$\xi(\omega, |\boldsymbol{Z}|, C) = \frac{4\omega^2 L}{1 - |\boldsymbol{Z}|^2\omega^2 C^2},$$

$$\psi(\omega, |\boldsymbol{Z}|, L, C) = \frac{|\boldsymbol{Z}|^2 (1 - 2\,\omega^2 LC)^2}{1 - |\boldsymbol{Z}|^2\omega^2 C^2}. \quad (11a,b)$$

From measurements of the impedance module $|\boldsymbol{Z}|_i$ at n points with a frequency $\omega_i = 2\pi f_i$ (specified for f_i), parameters R, L, C of the equivalent circuit shall be determined. The parameter to be adjusted is $\beta = C$, at the characteristics of the criterion function are deleted $\phi_{\xi\psi}(\Theta_1)$ containing the local minimum.

Uncertainties of new variables ξ and ψ depend on the impedance module $|\boldsymbol{Z}|$, on the frequency of $\omega = 2\pi f$ and on the correlation between them determined by the coefficient ρ. They are determined from the LPU uncertainty propagation law in the formula:

$$\begin{bmatrix} u^2(\xi) & \rho u(\xi)u(\psi) \\ \rho u(\xi)u(\psi) & u^2(\psi) \end{bmatrix} = \boldsymbol{G} \begin{bmatrix} u^2(\omega) & 0 \\ 0 & u^2(|\boldsymbol{Z}|) \end{bmatrix} \boldsymbol{G^T}, \quad (12)$$

where: $\boldsymbol{G}$ – is the Jacobian matrix of first derivatives – i.e. the sensitivity coefficients and is of the form

$$\boldsymbol{G} = \begin{bmatrix} \partial\xi/\partial f & \partial\xi/\partial|\boldsymbol{Z}| \\ \partial\psi/\partial f & \partial\psi/\partial|\boldsymbol{Z}| \end{bmatrix}. \quad (13)$$

The uncertainties of the new variables are:

$$u(\xi) = \frac{8\omega L}{(1 - |\boldsymbol{Z}|^2\omega^2 C^2)^2}\sqrt{u^2(\omega) + u^2(|\boldsymbol{Z}|)|\boldsymbol{Z}|^2\omega^6 C^4},$$

$$u(\psi) = \frac{2|\boldsymbol{Z}||1 - 2\,\omega^2 LC|}{\left(1 - |\boldsymbol{Z}|^2\omega^2 C^2\right)^2}[u^2(|\boldsymbol{Z}|)(1 - 2\,\omega^2 LC)^2 +$$

$$+u^2(\omega)\omega^2 C^2 |\boldsymbol{Z}|^2 (|\boldsymbol{Z}|^2 C(1 + 2\omega^2 LC) - 4L)^2]^{0,5}. \quad (14\text{ a,b})$$

The covariance part containing the correlation coefficient is of the form:

$$\rho u(\psi)u(\xi) = \frac{16(1 - 2\,\omega^2 LC)\omega^2|\boldsymbol{Z}|^2 LC}{(1 - |\boldsymbol{Z}|^2\omega^2 C^2)^4}[u^2(|\boldsymbol{Z}|)\omega^2 C(1 -$$

$$-2\,\omega^2 LC) + u^2(\omega)(|\boldsymbol{Z}|^2 C(1 + 2\omega^2 LC) - 4L)]. \quad (15)$$

Used in (4) effective inverse covariance matrix $\boldsymbol{U_{Yeff}^{-1}}$ is diagonal and the effective measurement uncertainty is given in the formula:

$$u_{eff}^2 = \theta_1^2 u^2(\xi) - 2\theta_1\,\rho u(\xi)u(\psi) + u^2(\psi). \quad (16)$$

The nominal values of the equivalent circuit parameters are as follows: L =1 nH, C = 350 pF, R =1 Ω, the data obtained from the measurements are presented in Table 1.

Table 1 – Values of measured frequencies and impedances at the measuring points

No	1	2	3	4	5	6	7	8	9	10
f, MHz	145	160	175	190	205	220	235	250	265	280
$\lvert\boldsymbol{Z}\rvert$, Ω	3.93	4.87	5.81	6.2	5.76	5.0	4.23	3.62	3,13	2.75

The standard frequency uncertainty is $u(f) = 1/\sqrt{3}$ Hz and the impedance module $|\boldsymbol{Z}|$ is measured with relative uncertainty $\delta(|\boldsymbol{Z}|) \approx 2$ %.

Standard uncertainties for three parameters $v = [R, L, C]$ estimate by law their propagation LPU

$$u^2(v) = \sum_{i=1}^{n} u^2(f_i)\left(\frac{\partial v}{\partial f}|_{f=f_i}\right)^2 + u^2(|Z|_i)\left(\frac{\partial v}{\partial |\boldsymbol{Z}|}|_{|Z|=|Z|_i}\right)^2 (17),$$

where the sensitivity coefficients $\frac{\partial v}{\partial f}$ and $\frac{\partial v}{\partial |Z|}$ are determined numerically.

As a result of parameter matching $\beta = C$ charts received $\phi_{\xi\psi}(\Theta_1)$ quasi-quadratic criterion function, where the global minimum is estimated approximately as $\phi_{\xi\psi_1 globalmin} \approx 0{,}182$ for $L \approx 0{,}992$ nH. It is being shown on Fig. 4.

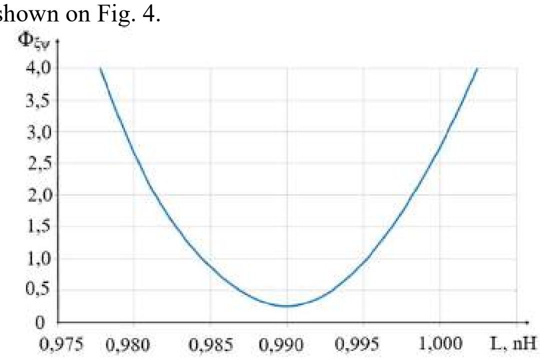


Fig. 4. Variable-dependent criterion function $\Theta_1 = L$

Parameter value $\beta = C$ corresponding to the global minimum of $C = 352$ pF. The matched value received $C = 0{,}985\ \Omega$.

Calculations of mean values of parameters are performed in EXCEL and in the R environment, and in addition, the covariance matrix, correlator matrix and uncertainty are obtained from a properly prepared script in R. From the numerical experiment the matrix $\boldsymbol{U_p}$ Matching parameters of the form:

$$\boldsymbol{U_p} = \begin{bmatrix} 1{,}91 \cdot 10^{-4}\Omega^2 & 4{,}09 \cdot 10^{-14}\Omega\text{H} & 1{,}03 \cdot 10^{-1}\ \Omega\text{F} \\ 4{,}09 \cdot 10^{-14}\Omega\text{H} & 4{,}3 \cdot 10^{-2}\ \text{H}^2 & 1{,}4\ 10^{-23}\text{HF} \\ 1{,}03 \cdot 10^{-14}\Omega\text{F} & 1{,}4\ 10^{-23}\text{HF} & 6 \cdot 10^{-24}\text{F}^2 \end{bmatrix} \quad (18)$$

and is bound to the correlator matrix

$$\boldsymbol{V} = \begin{bmatrix} 1 & 0{,}45 & 0{,}3 \\ 0{,}45 & 1 & 0{,}9 \\ 0{,}3 & 0{,}9 & 1 \end{bmatrix}. \quad (19)$$

From this correlator matrix, it follows that inductance and capacitance (0,9) are positively correlated, followed by inductance with resistance (0.45) and resistance with capacitance correlate weakly (0,3). Diagonal elements of a matrix $\boldsymbol{U_p}$ are squares of standard uncertainties and hence they follow:

R =0,985 (0,0138) Ω, (1,4%);
L =0,992(0,0066) nH (0,66%);
C =352 (2,44) pF (0,7%).

The uncertainty of standard parameters of the equivalent resistor scheme and their mutual correlations also results in the standard uncertainty of the frequency response corridor and the expanded uncertainty $u^2(|\boldsymbol{Z}|(\omega)) = \boldsymbol{S U_p S^T}$, $U(|\boldsymbol{Z}|(\omega)) = t_{1-\frac{\alpha}{2}, n-m} u(|\boldsymbol{Z}|(\omega))$.

Assumed is: α = 0,05, n = 10, m =3, $t_{1-\frac{0.05}{2}, 10-3}$ =2,36.

The elements of the vector were also determined, which are analytical partial derivatives of the impedance modulus and sensitivity coefficients $\boldsymbol{S} = \left[\frac{\partial |Z|}{\partial R}, \frac{\partial |Z|}{\partial L}, \frac{\partial |Z|}{\partial C}\right]$ with the following formulas:

$$\frac{\partial |\boldsymbol{Z}|}{\partial R} = \frac{R(1 - 4\omega^2 LC)}{Z'},$$

$$\frac{\partial |\boldsymbol{Z}|}{\partial L} = \frac{2\omega^2(2L - 4\omega^2 L^2 C + R^2 C)}{Z'},$$

$$\frac{\partial |\boldsymbol{Z}|}{\partial C} = -\frac{\omega^2(4\omega^2 L^2 + R^2)(R^2 C - 2L + 4\omega^2 L^2 C)}{Z'}, \quad (20a,b,c)$$

where

$$Z' = |\boldsymbol{Z}|[(1 - 2\,\omega^2 LC)^2 + \omega^2 R^2 C^2]^2. \quad (21)$$

Figure 5 shows respectively: measurement points, the nominal and matched frequency response of the impedance module $|\boldsymbol{Z}|$ with corridors of standard uncertainty $u(|\boldsymbol{Z}|)$ and expanded uncertainty $U(|\boldsymbol{Z}|)$.

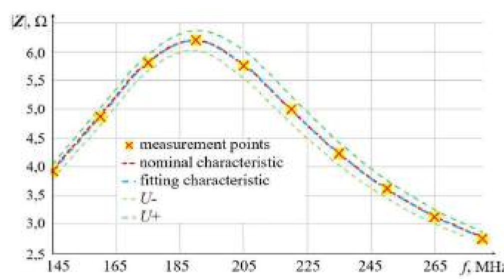


Fig. 5. Measuring points, nominal and matched impedance frequency responses

Figure 6 shows the fitting errors and the standard and expanded uncertainty corridors in relative units for the impedance modulus $|\boldsymbol{Z}|$ as a function of frequency with measurement points.

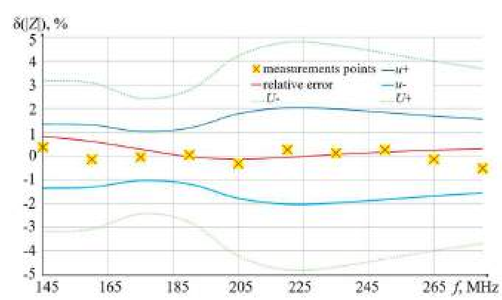


Fig. 6. Standard and expanded coverage corridor relative uncertainty and relative error

The uncertainty corridor of the impedance module $|\boldsymbol{Z}|$, over the entire range of characteristics under study varies from 2,5 % to less than 5 %. Its narrowest width occurs for the rising edge of the impedance modulus, i.e. for about 170 MHz, and the widest – for about 220 MHz, i.e. after reaching the maximum value for the falling edge of the characteristic.

4. Determination of capacitor model parameters from impedance components

The multi-element equivalent circuits are used in modeling and descriptions of frequency changes of components of capacitors' impedance. Such a scheme with three ideal resistances and two capacitances is on the figure 7.

Values R, C do not depend on the frequency. It is used for AC in the frequency range of 10 Hz to 10 GHz. Monte Carlo tests with it was made for the range 10Hz - 100 kHz in works [7-13].

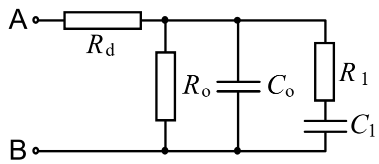


Fig. 7. Diagram of capacitor replacement circuit

Impedance $\boldsymbol{Z}$ the two ports A B for each frequency is described in complex numbers as:

$$\boldsymbol{Z} = Re\boldsymbol{Z} + jIm\boldsymbol{Z}, \quad (22)$$

where components: resistance $Re\boldsymbol{Z}$ and capacitive reactance $Im\boldsymbol{Z} = -1/\omega C$.

Impedance $\boldsymbol{Z}$ between two-terminals AB shall be measured directly or shall be determined from the voltage U_{AB}, current I_{AB} and the angle of their phase difference at each given frequency.

In our considerations parameters of the capacitor model were adjusted to the results of measurements of alternating current components with five frequencies using the least squares method. Then numbers of the measured points and their parameters are $n = 10$ and $m = 5$. The results are given in Table 2.

Table 2 – Measurement data of frequency and corresponding impedance components of the resistor and capacitor schemes

Lp	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
f, Hz	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}
$Re\boldsymbol{Z}$, kΩ	508,3	65,61	1,107	0,0113	0,0003	0,0002	0,0002	0,0002	0,0002	0,0002
C, nF|	8,476	6,006	5,015	5,000	5,0005	5,000	5,000	5,000	5,000	5,000

The capacitor equivalent circuit from Fig. 7 contains a resistance R_d connected in series with parallel connected resistance R_0, capacity C_0 and branch of connected in serial C_1 and R_1, i.e. of impedance

$$R_1 + \frac{1}{j\omega\, C_1} = \frac{1 + j\omega\, R_1\, C_1}{j\omega\, C_1}.$$

Hence

$$\frac{1}{Re\boldsymbol{Z} - R_d + j\, Im\boldsymbol{Z}} = \frac{1}{R_0} + j\omega\, C_0 + \frac{j\omega\, C_1}{1 + j\omega\, R_1\, C_1}. \quad (23)$$

After transforming the terms into complex numbers, i.e. after multiplying the denominators and numerators by conjugating expressions to their denominators, we get

$$\frac{Re\boldsymbol{Z} - R_d}{(Re\boldsymbol{Z} - R_d)^2 + Im\boldsymbol{Z}^2} - j\frac{Im\boldsymbol{Z}}{(R - R_d)^2 + Im\boldsymbol{Z}^2} =$$

$$= \frac{1}{R_0} + \frac{\omega^2\, C_1^2\, R_1}{1 + \omega^2\, R_1^2\, C_1^2} + j\omega\left(C_0 + \frac{C_1}{1 + \omega^2\, R_1^2\, C_1^2}\right). \quad (24)$$

The equality of the real and imaginary components of both sides of the expression (24) results in two equations:

$$\frac{Re\boldsymbol{Z} - R_d}{(Re\boldsymbol{Z} - R_d)^2 + Im\boldsymbol{Z}^2} = \frac{1}{R_0} + C_1\frac{\omega^2\ C_1 R_1}{1 + \omega^2\, R_1^2\, C_1^2},$$

$$-\frac{Im\boldsymbol{Z}}{\omega}\frac{1}{(Re\boldsymbol{Z} - R_d)^2 + Im\boldsymbol{Z}^2} = C_0 + C_1\frac{1}{1 + \omega^2\, R_1^2\, C_1^2}. \quad (25a,b)$$

The equations (25a,b) in the new variables are linear with the parameters $\beta_1 = R_1$ and $\beta_2 = R_d$, i.e.:

$$\psi_1(\beta_2) = \frac{1}{R_0} + C_1\xi_1(\beta_1)$$

and

$$\psi_2(\beta_2) = C_0 + C_1\xi_2(\beta_1) \quad (26a,b)$$

The abscissa and elevations in the new variables are as follows:

$$\xi_{1i} = \frac{\omega_i^2\ C_1\beta_1}{1 + \omega_i^2\beta_1^2\, C_1^2}$$

and

$$\xi_{2i} = \frac{1}{1 + \omega_i^2\beta_1^2\, C_1^2}\ ; \quad (27a,b)$$

$$\psi_{1i} = \frac{Re\boldsymbol{Z}_i - \beta_2}{(Re\boldsymbol{Z}_i - \beta_2)^2 + Im\boldsymbol{Z}_i^2}$$

and

$$\psi_{2i} = \frac{-Im\boldsymbol{Z}_i/\omega_i}{(Re\boldsymbol{Z}_i - \beta_2)^2 + I\ \ \boldsymbol{Z}_i^2}. \quad (28a,b)$$

From the law of error propagation follows the propagation of uncertainty according to LPU:

$$u(\xi_{1i}) = \frac{2C_1\beta_1\omega_i}{(1 + \omega_i^2\beta_1^2\, C_1^2)^2}u(\omega_i),$$

$$u(\xi_{2i}) = \frac{2C_1^2\beta_1^2\omega_i}{(1 + \omega_i^2\beta_1^2\, C_1^2)^2}u(\omega_i) = C_1\beta_1 u(\xi_{1i}),$$

$$u(\psi_{1i}) = \frac{1}{[(Re\boldsymbol{Z}_i - \beta_2)^2 + Im\boldsymbol{Z}_i^2]^2} *$$

$$* \{[[Im\boldsymbol{Z}_i^2 - (Re\boldsymbol{Z}_i - \beta_2)^2]^2 u^2(Re\boldsymbol{Z}_i) +$$

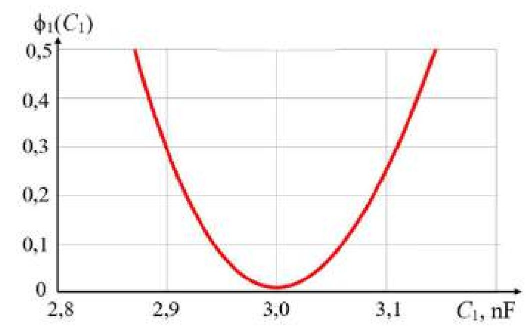
$$+ 4Im\boldsymbol{Z}_i^2(Re\boldsymbol{Z}_i - \beta_2)^2 u^2(Im\boldsymbol{Z}_i)\}^{0,5}; \quad (29a,b,c)$$

$$u(\psi_{2i}) = \frac{1}{\omega_i[(Re\boldsymbol{Z}_i - \beta_2)^2 + Im\boldsymbol{Z}_i^2]^2} *$$
$$* \{Im\boldsymbol{Z}_i^2[(Re\boldsymbol{Z}_i - \beta_2)^2 + Im\boldsymbol{Z}_i^2]^2 u^2(\omega_i)/\omega_i^2 +$$
$$+4(Re\boldsymbol{Z}_i - \beta_2)^2 Im\boldsymbol{Z}_i^2 u^2(Re\boldsymbol{Z}_i) +$$
$$+[(Re\boldsymbol{Z}_i - \beta_2)^2 - Im\boldsymbol{Z}_i^2]^2 u^2(Im\boldsymbol{Z}_i)\}^{0,5}. \quad (29d)$$

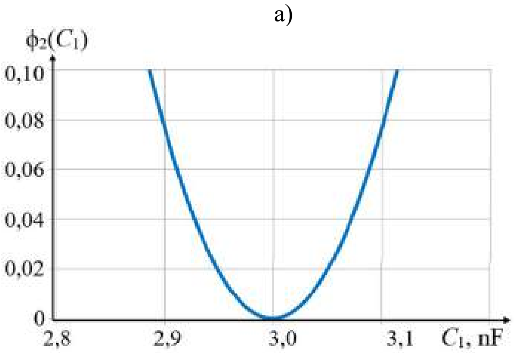
Assuming that measurements of both components are subject to uncertainties $\delta(Re\boldsymbol{Z}_i), \delta(Im\boldsymbol{Z}_i) < 1\%$ $(i = 1, \dots, n)$ and that the uncertainty of frequency measurement $u(f) < 0{,}58$ Hz, criterion functions are obtained $\phi_1(C_1)$, $\phi_2(C_1)$ shown in figure 8. They are determined from formula (4), for matrices inverse to effective covariance matrices for uncorrelated input quantities as $u_{eff}^2 = \theta_1^2 u^2(\xi) + u^2(\psi)$. New variables ξ_1, ψ_1 and ξ_2, ψ_2, despite, that ξ_2 and ψ_2 depend on the ω, they are practically uncorrelated.

The global minimum shall be obtained for $\beta_1 = R_1 = 1$ MΩ and $\beta_2 = R_d = 0{,}2$ Ω. Number of measurement points $n = 10$, and number of parameters $m = 5$. Received:

- minimum values of criterion functions $\phi_{1\min}(C_1) < 0{,}015$, $\phi_{2\min}(C_1) < 0{,}0001$;
- adjusted parameter values: $C_1 = 3$ nF, $C_0 = 5{,}000055$ nF, $R_0 = 9{,}999608$ MΩ.



a)



b)

Fig. 8. Graphs a) and b) of the criterion function for equations (26a) and (26b)

The uncertainties and correlations between quantities are due to the law of propagation of the uncertainty of the covariance matrix $\boldsymbol{U_p}$ quantities, i.e. for the parameters of the:

$$\boldsymbol{U_p}(C_1, R_0, C_0, R_d, R_1) = \boldsymbol{C}\boldsymbol{U_{in}}\boldsymbol{C}^T. \quad (30)$$

Diagonal matrix $\boldsymbol{U_{in}}$ has a size $3n$ x $3n$. For uncorrelated input quantities, it contains the following squares of uncertainty as elements: $u^2(f_i)$ for frequencies, $u^2(Re\boldsymbol{Z}_i)$ for the actual impedance component and $u^2(Im\,\boldsymbol{Z}_i)$ for the imaginary impedance component. Matrix $\boldsymbol{C}$ sensitivity ratios of size m x $3n$ is the matrix of Jacobian. It is obtained by numerical differentiation of all parameters $m = 5$ due to 3 x n input values. Symbolically, this is written as

$$C = \begin{bmatrix} \frac{\partial C_1}{\partial f_1} & \cdots & \frac{\partial C_1}{\partial f_n} & \frac{\partial C_1}{\partial Re|\boldsymbol{Z}|_1} & \cdots & \frac{\partial C_1}{\partial Re|\boldsymbol{Z}|_n} & \frac{\partial C_1}{\partial I\ \ |\boldsymbol{Z}|_1} & \cdots & \frac{\partial C_1}{\partial Im|\boldsymbol{Z}|_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial R_1}{\partial f_1} & \cdots & \frac{\partial R_1}{\partial f_n} & \frac{\partial R_1}{\partial Re|\boldsymbol{Z}|_1} & \cdots & \frac{\partial R_1}{\partial Re|\boldsymbol{Z}|_n} & \frac{\partial R_1}{\partial Im|\boldsymbol{Z}|_1} & \cdots & \frac{\partial R_1}{\partial Im|\boldsymbol{Z}|_n} \end{bmatrix}. \quad (30a)$$

Parameter covariance matrix $\boldsymbol{U_p}(C_1, R_0, C_0, R_d, R_1)$ is both sides product of the correlator $\boldsymbol{V}$ and matrix $\boldsymbol{Q}$ size of 5 x 5, i.e.:

$$\boldsymbol{U_P} = \boldsymbol{Q}^T\boldsymbol{V}\boldsymbol{Q}\,, \quad (30b)$$

where

$$\boldsymbol{Q} = \begin{bmatrix} u(C_1) & 0 & 0 & 0 & 0 \\ 0 & u(R_0) & 0 & 0 & 0 \\ 0 & 0 & u(C_0) & 0 & 0 \\ 0 & 0 & 0 & u(R_d) & 0 \\ 0 & 0 & 0 & 0 & u(R_1) \end{bmatrix}.$$

From this relationship, the values of the elements of the correlator matrix are obtained $\boldsymbol{V}$ described in the form of table 3.

Table 3 – Correlator matrix $\boldsymbol{V}$ data

	C_1	R_0	C_0	R_d	R_1
C_1	1	0,69	0,08	-0,013	0,22
R_0	0,69	1	-0,25	-0,02	0,64
C_0	0,08	-0,25	1	0,035	-0,46
R_d	-0,013	-0,02	0,035	1	-0,023
R_1	0,22	0,64	-0,46	-0,023	1

Table 3 shows that measurements of impedance components as a function of frequency lead to a strong positive correlation between the resistance R_0 and capacity C_1 (0.69), resistances R_0 and R_1 (0.64) and to the weak correlation between the capacity of the C_1 and resistance R_1 (0.22). A negative correlation occurs between the capacity of the C_0 and resistance R_1 (-0,46) and as weaker for capacity C_0 with resistance R_0 (-0,25). The other pairs of elements are practically uncorrelated.

The uncertainties of elements of the capacitor equivalent diagram are described by the formula:

$$u^2(w) = \sum_{i=1}^{n}[u^2(f_i)(\frac{\partial w}{\partial f}|_{f=f_i})^2 +$$
$$+u^2(Re\boldsymbol{Z}_i)(\frac{\partial w}{\partial Re\boldsymbol{Z}}|_{Re\boldsymbol{Z}=Re\boldsymbol{Z}_i})^2 +$$
$$+u^2(Im\boldsymbol{Z}_i)\left(\frac{\partial w}{\partial Im\boldsymbol{Z}}|_{Im\boldsymbol{Z}=Im\boldsymbol{Z}_i})^2\right], \quad (31)$$

where: $w = \{C_1, R_0, C_0, R_d, R_1\}$.

The results of measurements of the elements of the equivalent diagram along with their uncertainties are as follows:

$C_0 = 5{,}000055(0{,}017)$ nF (0,34%);
$R_0 = 9{,}9996\ (0{,}78)$ MΩ (7,8%);
$C_1 = 3(0{,}19)$ nF (6,33 %);
$R_1 = 1(0{,}016)$ MΩ (1,6 %);
$R_d = 0{,}2\ (0{,}0015)$ Ω (0,75%).

The standard uncertainty of the impedance module, i.e. the width of its coverage corridor, is calculated from the uncertainty propagation law as:

$$u^2\big(|\boldsymbol{Z}|(\omega)\big) = \boldsymbol{S}\boldsymbol{U_p}\boldsymbol{S^T}, \qquad (32)$$

where: sensitivity vector $\boldsymbol{S}$ size of 1 x m is derived analytically – see Appendix A.

The expanded uncertainty is described by the formula:

$$U\big(|\boldsymbol{Z}|(\omega)\big) = t_{1-\frac{\alpha}{2},n-m}u\big(|\boldsymbol{Z}|(\omega)\big). \qquad (33)$$

In the numerical experiment discussed here, the: α = 0,05, n = 10, m = 5, $t_{1-\frac{0{,}05}{2};10-5} = 2{,}57$.

The frequency characteristics of impedance on the logarithmic, nominal and fit scales, together with the measurement points and additionally relative errors, are given in Figures 9, 10 and 11. Figure 11 shows that the width of the aisle (expanded uncertainty) is slightly below 1% in the range 5·(10^2 – 10^8) Hz. At the beginning of the measurement range, the width is the largest and decreases from slightly above 5 % to 1 % in the range from 10 Hz to 500 Hz. In the range from 100 MHz to 1 GHz it increases from 1% to 2 % and up to 10 GHz it is at 2% of the value of the impedance module.

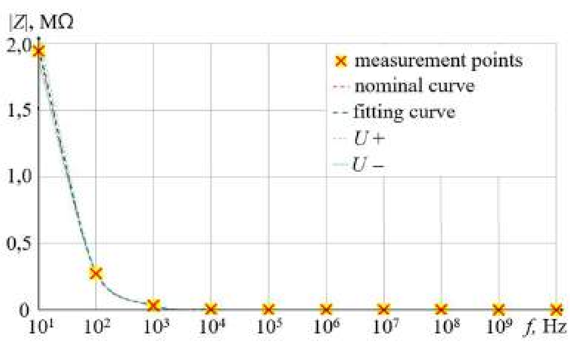


Fig. 9. Frequency impedance characteristics of the capacitor model with measurement points as nominal and fit curves (logarithmic scale on the frequency axis)

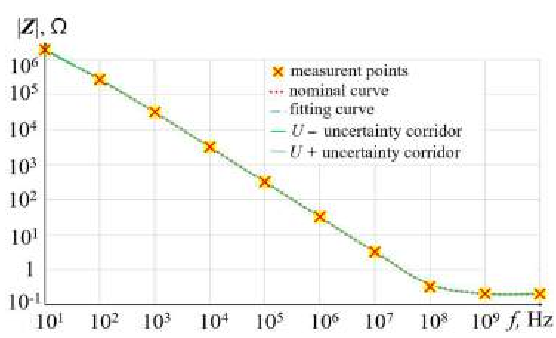


Fig. 10. Log-log frequency response of capacitor schematic impedance

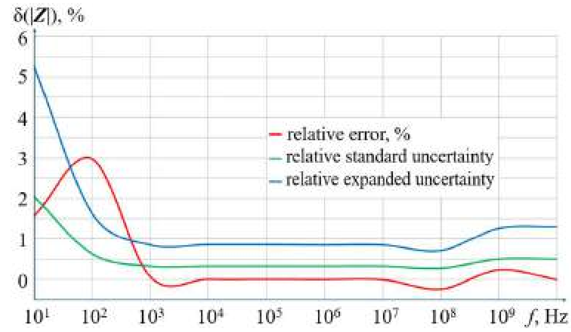


Fig. 11. Relative standard errors and the standard and expanded relative uncertainty of the impedance measurement of the capacitor equivalent diagram (logarithmic frequency scale).

5. Summary

This article introduces a method for fitting nonlinear curves to the data for measured points. It uses the change of variables to obtain linear relationships fitted according to the weighted least squares criterion of WTLS. After changing the variables in this way, you can also use straight line regression. The uncertainty of the coordinates of the measurement points is also considered, as well as their correlations, if they occur in the measurements.

The condition for using this method approximates the propagation of errors and measurement uncertainty, acceptable in metrology, using the first derivative of the transformation function. If the points under study are not too far from the nonlinear sought function, and the uncertainty of the data measurement is not too high, e.g. below 5%, then this method can be used successfully. The limitations are therefore the same as estimating the accuracy of measurements by the international GUM guide [1].

The method also allows to determine the uncertainty corridor for a nonlinear function fitted to the measurement data. It has already been used in the authors' papers [14-20] for several different examples of measurements with the change of one and both coordinates of a nonlinear function. Computational examples of fitting various nonlinear functions to given measurement points, including implicit functions, are presented. They showed that the method of changing variables is universal if the new variables are properly selected.

In literature, e.g. [2-6], simple examples of linearization of the function describing measurements are usually considered. There was no discussion of the method of fitting nonlinear functions with linearization by changing variables, nor a discussion of how to determine the boundaries of their uncertainty band without correlation and with correlations.

The examples considered in this paper use the results of measurements of the impedance module of resistor and capacitor equivalent circuits in n=10 frequencies. From them, the frequency characteristics of the impedance module were determined, nominal and adjusted to the parameters of the system using the WTLS method along with its uncertainty corridor. For the capacitor, the resistive and reactance components of the impedance of its equivalent scheme were matched and their uncertainties were determined using the law of their propagation, i.e. the LPU method.

The method used in this work for nonlinear functions can be fully useful in measurement practice. It is also worth considering the possibility of using it in the internationally developed extended version of the GUM guide.

References

1. JCGM 100:2008 Evaluation of Measurement Data. Guide to the Expression of Uncertainty in Measurement (Sevres, France: International Bureau of Weights and Measures BIPM), and its Supplements.

2. Levenberg K. (1944). A Method for the Solution of Certain Non-Linear Problems in Least Squares // Quarterly of Applied Mathematics. 2 (2) 164-168 doi:10.1090/qum/10666.

3. Marquardt D. (1963). An Algorithm for Least-Squares Estimation of Nonlinear Parameters // SIAM Journal on Applied Mathematics. 11 (2): 431–41 doi:10.1137/0111030.

4. Dennis J.E., Jr., Schnabel R.B. (1983). Numerical Methods for Unconstrained Optimization and Nonlinear Equations // SIAM 1996 reproduction of Prentice-Hall 1983.

5. Draper R.D., Smith H., (1998) Applied Regression Analysis 3rd Ed., J. Willey, New York.

6. Madsen K., Nielsen H.B., Tinglef (2004), Method for Non-Linear Least Squares Problems, 2nd Edition. April 2004. Technical University of Denmark.

7. Kubisa S., Warsza Z. Uogólniony opis właściwości częstotliwościowych rezystorów // Elektronika, 3/2016 s.31-38 DOI: 10.15199/13.2016.3.1.

8. Kubisa S., Warsza Z.L. Analiza błędów częstotliwościowych rezystorów. Część 1. Modele rezystorów przy prądzie przemiennym i ich parametry // Przegląd Elektrotechniczny 6/2016 s. 211-216.

9. Kubisa S., Warsza Z.L. Analiza błędów częstotliwościowych rezystorów. Część 2. Korekcja częstotliwościowa impedancji rezystorów i jej skuteczność // Przegląd Elektrotechniczny 7/2016, 197-201.

10. Kubisa S., Warsza Z. L. The frequency analysis of real resistors in relative values // Measurement Automation Monitoring (MAM 2016) vol. 62, no 03, s. 80-86.

11. Kubisa S., Warsza Z. L. Generalized description of the frequency characteristics of resistors // SCIT 2016 Recent Advances in Systems, Control, and Information Technology. (Editors: R. Szewczyk et all), vol. 543, series: Advances in Intelligent Systems and Computing. Springer Int. Publ.2017, 630-644, DOI 10.1007/978-3-319- 48923-0_67.

12. Kubisa S., Warsza Z. L. Identification of parameters of the capacitor equivalent scheme using Monte Carlo methods. In: R. Szewczyk et all (ed.) // Automation 2017 Innovations in Automation, Robotics and Measurement Techniques, series Advances in Intelligent Systems and Computing 550. Springer Int. Publishing AG 2017 p.166-168. DOI 10.1007/978-3-319-54042-9-15.

13. Kubisa S., Warsza Z. Dokładność identyfikacji parametrów modelu kondensatora 2 metodami MonteCarlo. PAR 2_2018 s. 41-48.

14. Puchalski J., Warsza Z.L.: The method of fitting a non-linear function to data of measured points and its uncertainty band. (*in Polish*: Estymacja niepewności wybranych funkcji nieliniowych wyznaczanych z pomiarów metodą regresji liniowej) // Pomiary Automatyka Robotyka PAR n. 3_2023 p. 45 –55, DOI:10.14313/PAR249/45.

15. Puchalski J.G. Nonlinear Curve Fitting to Measurement Points with WTLS Method Using Approximation of Linear Model // Int J Auto AI Mach Learn. Vol. 4 Issue 1, June 2024 pp.36-60. ISSN 2593-7568.

16. Warsza Z.L., Puchalski J. Assessment of the uncertainty of selected nonlinear functions determined from measurements by the linear regression method. Monography: Advanced Mathematical and Computational Tools in Metrology and Testing XIII. Pavese F., Forbes A.B., A Bošnjaković et all (eds.): Series on Advances in Mathematics for Applied Sciences. v. 94 © **2025** World Scientific Publishing Co. N. Jersey ·London ·Singapore, pp.288–307, doi.org/ 10.1142/ 9789819800674_0027.

17. Warsza Z.L., Puchalski J., Więcek T.: Method of fitting a nonlinear function to measurement data and its uncertainty band // Proceedings of 15th scientific conference SP'2024 S: Measurement systems in research and industry. Łagów 9-12.06.2024 University of Zielona Gora 2024 (Uniwersytet Zielonogórski) pp.151-154. 978-83-957716-4-4

18. Warsza Z.L., Puchalski J., Więcek T.: Method with changing variables for fitting a nonlinear function to measurement data and its uncertainty band. // Measurement Sensors 38(2):101730. DOI:10.1016/j.measen.2024.101730

19. Warsza Z.L., Puchalski J., Więcek T.: Metoda ze zmianą zmiennych dla dopasowania funkcji nieliniowych do danych pomiarowych i ich niepewności (Konf. PPM 2024 Gliwice) Przegląd Elektrotechniczny, ISSN 0033-2097, R. 100 n. 12/2024 s.252 -259 doi:10.15199/48.2024.12.55

20. Warsza Z.L., Puchalski J., Więcek T.: Przykłady dopasowywania charakterystyk nieliniowych metodą zmiany zmiennych i ich niepewności. Metrologia red. M.R Rząsa (monografia konferencji: 56 MKM Kalisz). Politechnika Opolska, Opole 2024 s. 29-44. ISSN 1429-6063, ISBN 978-83-66903-71-5.

21. Warsza Z. L., Puchalski J., Więcek T.: Novel method of fitting a nonlinear function to measurement data based on linearization by change variables, examples and uncertainty // Open access Journal Metrology MDPI 2024, 4(4), 718-735; https://doi.org/10.3390/metrology4040042

22. Puchalski J., Warsza Z.L., Wyznaczanie parametrów schematów zastępczych rezystora i kondensatora z pomiarów częstotliwościowej charakterystyki ich impedancji // LVII Międzyuczelniana Konferencja Metrologów Poznań 22-24. 09. 2025 Materiały konferencyjne. Wydawnictwo Politechniki Poznańskiej. Poznań 2025 s. 325 -350

Supplement

Below are the analytically determined partial derivatives of the capacitor's equivalent impedance modulus for all parameters of the equivalent schematic elements. Impedances are represented as a series combination of resistances R_d and complex impedance $\frac{1}{A+jB}$:

$$\boldsymbol{Z} = \frac{1}{A+jB} + R_d, \qquad \text{(A. 1)}$$

where

$$\mathrm{A} = \frac{1}{\mathrm{R}_0} + \frac{\mathrm{R}_1}{\mathrm{R}_1^2 + \frac{1}{\omega^2 \mathrm{C}_1^2}} = \frac{1}{\mathrm{R}_0} + \frac{\omega^2 \mathrm{C}_1^2 \mathrm{R}_1}{\omega^2 \mathrm{C}_1^2 \mathrm{R}_1^2 + 1} \text{ and } B = \omega\left(C_0 + \frac{C_1}{\omega^2 C_1^2 R_1^2 + 1}\right) \qquad \text{(A.2) and (A.3)}$$

The partial derivative with respect to R_d for impedance square $Z^2 (\mathrm{Z} = |\mathbf{Z}|)$, i. e. $\frac{\partial Z^2}{\partial R_d} = 2Z\frac{\partial Z}{\partial R_d}$, where

$$\frac{\partial Z}{\partial R_d} = \frac{1}{2Z}\frac{\partial Z^2}{\partial R_d} = \frac{1}{Z}\left(\frac{A}{A^2+B^2} + R_d\right) \qquad \text{(A.4)}$$

and similarly, derivatives with respect to R_0, C_0 i R_1:

$$\frac{\partial Z}{\partial R_0} = \frac{1}{2Z}\frac{\partial Z^2}{\partial R_0} = \frac{1}{ZR_0^2}\frac{A+(A^2-B^2)R_d}{(A^2+B^2)^2}\ ; \qquad \text{(A.5)}$$

$$\frac{\partial Z}{\partial C_0} = \frac{1}{2Z}\frac{\partial Z^2}{\partial C_0} = -\frac{\omega AB}{Z(A^2+B^2)^2}\left(\frac{1}{A} + 2R_d\right); \qquad \text{(A.6)}$$

$$\frac{\partial Z}{\partial R_1} = \frac{1}{2Z}\frac{\partial Z^2}{\partial R_1} = \frac{\omega^2 C_1^2\left(R_d\left(\left(B^2-A^2\right)\left(1-\omega^2 C_1^2 R_1^2\right)+4AB\omega\ _1 R_1\right)-A\left(1-\omega^2 C_1^2 R_1^2\right)+2B\omega C_1 R_1\right)}{Z(A^2+B^2)^2(\omega^2 C_1^2 R_1^2+1)^2}. \qquad \text{(A.7)}$$

Derivation of the derivative with respect to C_1:

$$\frac{\partial Z}{\partial C_1} = \frac{1}{2Z}\frac{\partial Z^2}{\partial C_1} = \frac{2\omega C_1 R_1(R_d(B^2-A^2)-A)-(2ABR_d+B)\left(1-\omega^2 C_1^2 R_1^2\right)}{Z(A^2+B^2)^2(\omega^2 C_1^2 R_1^2+1)^2}. \qquad \text{(A.8)}$$

Надійшла (Received) 11.10.2025
Прийнята до друку (accepted for publication) 11.11.2025

ВІДОМОСТІ ПРО АВТОРІВ/ABOUT THE AUTHORS

Варша Зігмунд – PHD, Польське метрологічне товариство, Варшава, Польща, e-mail: zlw1936@gmail.com ORCID:0000-0002-3537-6134
Warsza Zygmunt – PHD, Polish Metrological Society, Warsaw, Poland, e-mail: zlw1936@gmail.com ORCID:0000-0002-3537-6134

Пухальський Яцек – PHD, головний метролог Центральне управління мір GUM, Варшава, Польща, e-mail: j.puchalski@gum.gov.pl, ORCID: 0000-0002-5055-8550
Puchalski Jacek – PHD, chief metrologist of Central Office of Measures GUM, Warszawa, Poland, e-mail: j.puchalski@gum.gov.pl, ORCID: 0000-0002-5055-8550

Визначення параметрів еквівалентних схем резисторів та конденсаторів за вимірюванням частотних характеристик їх імпедансу

Зигмунд Л. Варша, Яцек Пухальський

Анотація
У статті представлено метод визначення параметрів еквівалентних схем резисторів та конденсаторів шляхом з'єднання двовивідних елементів з одиничним лінійним опором, індуктивністю або ємністю. Значення та невизначеності цих параметрів оцінюються за допомогою методу найменших квадратів для вимірювань частотної характеристики модуля імпедансу. Це завдання є математично складним, оскільки зазвичай отримується система нелінійних рівнянь, яка аналітично не розв'язується. Для отримання лінійних рівнянь було запропоновано використовувати метод заміни змінних. Цей метод був раніше розроблений авторами для регресії нелінійних функцій і вже успішно використовується в метрологічних задачах.

Ключові слова: схеми заміщення, опір, ємності, індуктивність, частотна характеристика, модуль імпедансу.

